

Development of a statistical tool for the estimation of riverbank erosion probability

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Abstract

Riverbank erosion affects river morphology and local habitat, and results in riparian land loss, property and infrastructure damage, and ultimately flood defence weakening. An important issue concerning riverbank erosion is the identification of the vulnerable areas in order to predict river changes and assist stream management/restoration. An approach to predict areas vulnerable to erosion is to quantify the erosion probability by identifying the underlying relations between riverbank erosion and geomorphological or hydrological variables that prevent or stimulate erosion. In the present work, a statistical methodology is proposed to predict the probability of presence or absence of erosion in a river section. A physically based model determines the locations vulnerable to erosion by quantifying the potential eroded area. The derived results are used to determine validation locations for the evaluation of the statistical tool performance. The statistical tool is based on a series of independent local variables and employs the Logistic Regression methodology. It is developed in two forms, Logistic Regression and Locally Weighted Logistic Regression, which both deliver useful and accurate results. The second form though, provides the most accurate results as it validates the presence or absence of erosion at all validation locations. The proposed tool is easy to use, accurate and can be applied to any region and river.

1 Introduction

Riverbank erosion is a complex phenomenon resulting from various factors, which affect the balance of ecosystems. It is also important from the geomorphological aspect as it also induces

1 changes in the river channel course and in the development of the floodplain (Hooke, 1979;
2 Bridge, 2003). Mass-failure processes constitute a significant source of sediment in disturbed
3 streams, which occur due to a combination of hydraulic and geotechnical processes that
4 undercut bank toes and cause bank collapse (Simon et al., 2009). Riverbank erosion is a natural
5 geomorphologic process that affects the fluvial environment in many aspects; physical,
6 ecological and socio-economical. It is the result of a complex interaction between the channel
7 hydraulic conditions and the physical characteristics of the banks, both of which are highly
8 variable in nature. Bank retreat affects the riverbed structure and morphology as well as the
9 floodplain morphology and the physical habitat. In addition, riparian land losses and damage to
10 human property and infrastructures, lead to direct financial consequences. Moreover, turbidity
11 increase, sediment and debris transport, and flood defense weakening, reveal a complex
12 combination of arising issues due to riverbank erosion. According to Atkinson et al. (2003),
13 significant parameters affecting erosion are vegetation index (stability), the presence or absence
14 of meanders, bank material (classification) and stream power. Also other factors such as bank
15 height, riverbank slope, river cross section width, riverbed slope and water velocity have been
16 reported to affect the erosion rate (Hooke, 1979; Abam, 1993; Winterbottom and Gilvear, 2000;
17 Rinaldi et al., 2008; Luppi et al., 2009). Therefore, the identification of riverbanks which are
18 vulnerable to erosion is of utmost importance, either for their protection or restoration.

19 On the other hand, riverbank erosion constitutes a significant factor to the functioning of river
20 dependent ecosystems and provides a sediment source that creates riparian habitat. Bank
21 erosion is a key geomorphological mechanism for the fluvial ecosystems, since it regulates the
22 diversity of habitats, species and vegetal units. The process provides riparian vegetation
23 succession and develops dynamic habitats, vital for fluvial plants and animals. For small scale
24 bank erosion or for local extent, there is no significant influence on the aquatic ecosystem and
25 it is contributing to the ecosystem sustainability. In the opposite case, the ecosystem is
26 significantly affected, while riparian land losses and damages are caused providing areas
27 vulnerable to flooding (Piégay et al., 1997; Piégay et al., 2005; Florsheim, 2008).

28 The bank erosion process is closely related to soil composition of the riverbanks, and the
29 erodibility factor is affected by the composition of sand, silt and clay. A high content of sand
30 and silt leads to easily eroded soils since they are fine in size and can be carried away by river
31 flow. The most common type of bank structure is a stratified or interbedded bank of cohesive
32 or non-cohesive layers. Riverbanks made up of non-cohesive soil are very erodible due to the

1 low clay-content and the weak erosion-resistant strength of the bank soil. Instead cohesive soils
2 have increased clay or clayey silt content and are more resistant to erosion. Non-cohesive soils
3 erode as individual grains, while cohesive soils erode as aggregates. On the other hand a
4 bedrock bank is usually very stable and will only experience gradual erosion (Raudkivi 1998,
5 Roslan et al., 2013).

6 Although riverbank erosion is a common phenomenon, the prediction of the location and of the
7 extent of riverbank erosion is difficult. Therefore, a range of approaches and methods have been
8 developed and tested. The most important issue concerning riverbank erosion is the
9 identification of the areas vulnerable to bank erosion, in order to predict changes in the river
10 channel form and assist stream management/restoration options. Different methods have been
11 used to predict erodibility, such as analyses of historical maps and the use of sequential aerial
12 photographs based on GIS technology. However, riverbank erosion is usually approached by
13 using a combination of bank stability methods and hydrodynamic models to predict the
14 vulnerable areas and estimate the erosion rate (Nardi et al., 2013). Of these two methods, the
15 former has a relatively high degree of inaccuracy, while the latter is too complex to be applied,
16 as it requires significant number of data variables.

17 Herein, a statistical tool is proposed using the Logistic Regression (LR) technique, for the
18 determination of riverbank erosion probability. This technique was selected due to its ability to
19 link related dependent and independent variables, by converting their relationship to a
20 probability of presence or absence of the dependent variable. In addition, it can be extended to
21 account for locally spatial correlated independent variables. The suggested statistical model
22 entitled Locally Weighted Logistic Regression (LWLR), combines LR and Locally Weighted
23 Regression (LWR) principles to create a local model that calculates the probability of erosion
24 to occur based on spatially correlated secondary information (e.g. bank slope, river cross
25 section). Therefore, the accuracy of the predictions is expected to improve compared to the
26 global regression model LR.

27 The proposed statistical model identifies the underlying relations between riverbank erosion
28 and the geomorphological or hydrological variables that prevent or stimulate erosion. It utilises
29 the available data to detect areas vulnerable to erosion. In addition, the erosion occurrence
30 probability can be calculated in conjunction with the model deviance for each independent
31 variable or model form tested. A similar method was introduced and applied successfully to a

1 [river in North Wales \(Atkinson et al., 2003\), for the estimation of the variables that mostly](#)
2 [affect riverbank erosion. The simple Logistic Regression was applied.](#)

3 This work also involves the application of the Bank-Stability and Toe-Erosion Model (BSTEM
4 5.2) in order to predict eroded or not riverbank areas, for the validation of the proposed tool.
5 The BSTEM model is a physically-based model, developed by the National Sedimentation
6 Laboratory in Oxford, Mississippi, USA (Simon et al., 2000), and it has been used to simulate
7 the hydraulic and geotechnical processes responsible for mass failure. It represents two distinct
8 processes namely, the failure by shearing of a soil block of variable geometry and the erosion
9 by flow of bank and bank toe material. The BSTEM has been successfully applied in diverse
10 alluvial environments (e.g., Simon et al., 2000; Simon et al., 2002; Simon and Thomas, 2002;
11 Pollen and Simon, 2005; Pollen-Bankhead and Simon, 2009; Simon et al., 2011). [It](#) was used
12 to simulate the effects of enhanced matric suction from evapotranspiration and decreased soil
13 erodibility driven by the presence of plant roots, quantifying the effects on streambank factor
14 of safety and comparing with the effects of mechanical root-reinforcement (Pollen-Bankhead
15 and Simon, 2010). BSTEM was also used to quantify bank retreat, which ranged from 7.8 to
16 20.9 m along 100 m of riverbank at the Barren Fork Creek site (Midgley et al., 2012). [In](#)
17 [addition, it](#) was also used to quantify the reductions of mass failure frequency and sediment
18 loading from streambanks in the Lake Tahoe in United States (Simon et al., 2009).

19 [The developed methodology was applied to Koiliaris River Basin at the island of Crete, Greece.](#)
20 [The overall](#) concept of this work is to [provide estimates](#) of the erosion probability at specific
21 ungauged riverbank locations, [based on](#) independent secondary explanatory information [in](#)
22 [terms of LWLR methodology](#). BSTEM has an auxiliary role to estimate/validate potential
23 eroded riverbank locations by calculating the potential eroded area, using field measurements
24 of hydraulic, hydrologic and geomorphologic variables. These estimations (dependent
25 variables) are then used to set up and validate the statistical model. To the best of our
26 knowledge, the combination of deterministic and stochastic models to predict river bank erosion
27 appears for the first time in the scientific literature.

28

29 **2 Case Study**

30 The Koiliaris River Basin is situated 25 km east of Chania (005-12-489E, 039-22-112N) and
31 occupies an area of about 130 km². Watershed elevation ranges from 0 to 2041 m.a.s.l. with
32 slopes ranging from 1-2% at low elevations up to 43% (high elevations). [The total length of it's](#)

1 hydrographic network is 36 km (Moraetis et al., 2010). The area has been studied extensively
2 in the last ten years and especially since 2009 as part of the European network of Critical Zone
3 Observatories (Koiliaris CZO). The Koiliaris River Basin, as a typical Mediterranean
4 watershed, is characterized by varying spatial and temporal hydrologic and geochemical
5 processes. Lithology and geomorphology as well as the climatic conditions in the area, have
6 major influence on the hydrologic characteristics of the Koiliaris CZO (Moraetis et al., 2014).
7 The river is mainly fed by the Stylos karstic springs with water originating from the White
8 Mountains and traveling through an extensive karstic system, which drains the rain and snow
9 melt at high elevations. It is also fed temporarily, during the rain period (October to April), by
10 the Keramianos tributary stream. Keramianos is the main temporary tributary, which drains a
11 watershed sub-catchment characterized by steep slopes, schist geologic formation and degraded
12 erodible soils. As a result, when high rainfall intensities fall upon this area, especially after the
13 dry summer period, surface runoff is induced, transferring large quantities of sediments to the
14 Koiliaris River (flush floods) (Moraetis et al., 2010). During these events, river flow conditions
15 change dramatically with a rapid increase in water level and high flow velocities, affecting
16 riverbank erodibility up to causing bank failure. Such events occur two to three times a year
17 during the rainy period affecting the riparian area and enhancing soil losses through riverbank
18 erosion. The current study focuses on the downstream section of River Koiliaris (Fig. 1). A
19 hydrochemical station (Gauge Station), has been strategically located at the intersection of the
20 Koiliaris River with the Keramianos tributary, recording the water level which was used to
21 generate the flow hydrograph (Fig. 2).

22

23 **3 Methodology**

24 The bank erosion vulnerability of the Koiliaris' riverbanks was first studied during the
25 hydrologic period 2010-11. The downstream section of the river was divided in eight
26 subsections of variable length, starting from the Gauge Station up to pin No 8 on the study area
27 map (Fig. 1). In each subsection, the geomorphological characteristics of the riverbanks and the
28 riverbed were measured at the beginning and at the end of the subsection, during the first field
29 campaign. Channel and bank geometry characteristics, flow parameters, bank material, bank
30 vegetation and protection parameters were identified and used as an input to the BSTEM model
31 to calculate the riverbank eroded area (L^2).

1 Therefore, reach slope varied between 0.0042 and 0.11 m/m and the bank material was set, after
2 field measurements analysis, to “fine rounded sand” with an average medium grain size of 0.3
3 (± 0.06) mm. The “geyer willow” was selected from the predefined list to describe the bank
4 vegetation with the assumptions of the plants age of about 100 years and 100% contribution to
5 assemblage. Additionally, for the locations where the bank was protected, the “boulders” choice
6 was used to describe the bank material. Bank slope and river cross-section measurements were
7 supplemented by a second field campaign. As far as the flow parameters, river water elevation
8 was set to 1.27 m for a 48 h duration event, based on field data. The BSTEM model was then
9 applied to determine the vulnerability of bank erosion at the under study river subsections. The
10 model results, for such long distances (min = 20 m and max = 200 m), were interpreted as
11 potential erosion vulnerability of riverbank, considering the extent of the estimated eroded area.
12 At the beginning of hydrological year 2013-14, a second field campaign was designed to
13 identify this time specific locations vulnerable to erosion. Twelve riverbank locations were
14 selected along the aforementioned eight subsections and scaled sticks were installed. Two of
15 those locations were selected at restored parts of the river section to monitor potentially stable
16 riverbank points. Six months later, at the end of the wet period and after three flood events (Fig.
17 2 – Red peaks), the erosion sticks were visually inspected, during a field trip, to identify the
18 presence or absence of erosion. Therefore, the eroded area was roughly estimated.
19 The concept for this second campaign was to establish measurement points, necessary to
20 develop and apply a statistical model that taking into account a series of explanatory variables,
21 would determine the probability of riverbank erosion at local scale. Furthermore, a series of
22 validation points were necessary to validate the model's efficiency. Thus, the endpoints of each
23 subsection from the first campaign were used because an overall estimate of the riverbank
24 vulnerability was available from the BSTEM results.
25 However, in order to verify the BSTEM prediction efficiency, it was decided to test the model
26 by using the twelve locations of the second campaign. Based on the model's efficiency and the
27 quality of estimation, the reliability of BSTEM results was evaluated at the eight subsections of
28 the aforementioned river section. The 2nd BSTEM model application estimated the cumulative
29 riverbank erosion effect for the three flash flood events (Fig. 2) at the twelve locations. The
30 other parameters were similar for both model applications, since the same river section was
31 employed.

1 [The BSTEM model results \(at the twelve locations\) together with field inspection were used to](#)
2 [setup the statistical model by interpreting the erosion existence in terms of binary data \(1 =](#)
3 [“presence of erosion” and 0 = “no erosion”\).](#) The BSTEM model, has the capacity to
4 quantitatively calculate the eroded area (L^2). The interpretation of the significance of the
5 estimated eroded area was determined through a statistical process that involves the 25th and
6 75th percentiles of the estimated values. Therefore, the eroded area can be classified to [levels of](#)
7 [significance](#). Under the 25th percentile the erosion is categorized as not significant ([no erosion](#))
8 and over 75th as significant. The [in-between](#) values are signified as erosion. [The latter two, fall](#)
9 [in the ‘presence of erosion’ category.](#)

10 [Next,](#) the probability of erosion at the riverbanks of [River Koiliaris,](#) was estimated considering
11 a series of easy to determine independent geomorphological variables ([bank slope, river cross](#)
12 [section\) through LR and LWLR methodologies, first at the validation points and then at](#)
13 [ungauged riverbank locations.](#) [The methodological steps of the proposed tool and of the overall](#)
14 [process are briefly described by a flowchart presented in Figure 3.](#)

15 **3.1 Logistic Regression**

16 Riverbank erosion can be simulated by a regression model using independent variables that are
17 considered to affect the erosion process. The impact of such variables may vary with
18 geographical location and, therefore, a spatially non-stationary regression model is preferred
19 instead of a stationary equivalent. Locally Weighted Regression (LWR) is proposed as a
20 suitable choice. This method can be extended to predict the binary presence or absence of
21 erosion based on a series of independent local variables by using the Logistic Regression (LR)
22 model. It is referred to as Locally Weighted Logistic Regression (LWLR). The two independent
23 variables considered herein were river cross section width and bank slope.

24 In statistics, LR is a type of regression analysis used for predicting the outcome of a categorical
25 dependent variable (e.g. binary response) based on one or more predictor variables (continuous
26 or categorical). The method can be used along with LWR to assign weights to local independent
27 variables. LWR allows model parameters to vary over space in order to reflect spatial
28 heterogeneity (Atkinson et al., 2003; Lall et al., 2006). The probabilities of the possible
29 outcomes are modelled as a function of independent variables using a logistic function. LR
30 measures the relationship between a categorical dependent variable and, usually, one or several
31 continuous independent variables by converting the dependent variable to probability scores.

1 Then, a LR is formed, which predicts success or failure of a given binary variable (e.g. 1 =
2 “presence of erosion” and 0 = “no erosion”) for any value of the independent variables.

3 The LR model is based on the logistic function, a common sigmoid function. The mathematical
4 form is represented by the following equation:

$$5 \quad p(\mathbf{x}) = \frac{1}{1 + \exp(-\mathbf{x})}, \quad (1)$$

$$6 \quad \mathbf{x} = \beta_0 + \sum_{k=1}^K \beta_k x_k, \quad k = 1, \dots, K, \quad (2)$$

7 where $p(\mathbf{x})$ is the probability of the dependent variable, $0 \leq p(\mathbf{x}) \leq 1$, associated with a given
8 location, K is the number of the respective independent variables, $\beta_0, \beta_k, k = 1, \dots, K$ are the
9 logistic regression coefficients estimated from n sample observations and x_k are the
10 independent variables (Menard, 2001; Atkinson et al., 2003; Ozdemir, 2011). The regression
11 coefficients are estimated by using maximum likelihood estimation.

12 The goal of LR is to derive estimates for the $K + 1$ unknown parameters, $\beta_0, \beta_1, \dots, \beta_K$ by
13 maximizing the likelihood function given in Eq. (3):

$$14 \quad L(\boldsymbol{\beta} / y_1, \dots, y_n) = \prod_{i=1}^n (p(\mathbf{x}_i))^{y_i} (1 - p(\mathbf{x}_i))^{1 - y_i}, \quad (3)$$

15 where n is the sample size, \mathbf{x}_i represents the values of the independent variables for the i^{th}
16 sample (Eq. 2), $p(\mathbf{x}_i)$ is determined by Eq. (1) and y_i is the value of the dependent variable for
17 the i^{th} sample. As the equations are non-linear, the solution was numerically estimated using
18 Newton’s method (Hosmer and Lemeshow, 2004).

19 LWR is an extension to the concept of general regression. The difference between LWR and
20 Multiple Linear Regression is that in LWR the independent variables’ effect on the dependent
21 one is weighted based on a weighted function in terms of their geographical location. Basically,
22 LWR is a form of spatial data analysis that allows for the evaluation of a dependent variable
23 based on one or more local independent variables (Cleveland and Devlin, 1988; Brunson et
24 al., 1996; Fotheringham et al., 2002; Atkinson et al., 2003; Lall et al., 2006). LWR is used to
25 improve the results obtained with simple LR, allowing for the coefficients β_k to vary for each
26 estimation point. In this work, the exponential (Eq. 4) and the tri-cubic (Eq. 5) weighting
27 functions are used to assign weights to the observation points. The first was applied in a similar

1 work (Atkinson et al., 2003), while the latter is a common, efficient weighting function that is
 2 used with LWR.

$$3 \quad w(d) = \exp(-d/a), \quad (4)$$

$$4 \quad w(d) = \left[1 - |d/h|\right]^3, \quad |d/h| \leq 1. \quad (5)$$

5 In Eqs. (4) and (5) above, w denotes the weights, a and h are nonlinear parameters which
 6 determine the spatial correlation distance of measurement points with respect to the estimation
 7 point, for each function and d is the Euclidean distance between the estimation point and the
 8 measurement point.

9 **3.2 Calculation of model deviance**

10 The erosion occurrence probability can be calculated in conjunction with the model deviance.
 11 The reliability of both LR and LWLR is determined using the G-Statistic method. It is a simple
 12 and effective statistical approach to evaluate the model efficiency and the reliability of each of
 13 the independent variables tested. The model deviance is given by

$$14 \quad D = -2 \sum_{i=1}^n \left[y_i \ln \left(\frac{p(\mathbf{x}_i)}{y_i} \right) + (1 - y_i) \ln \left(\frac{1 - p(\mathbf{x}_i)}{1 - y_i} \right) \right], \quad (6)$$

15 where y is a binary variable that indicates the result of an experiment. The conditional
 16 probability of the effect to be present is expressed as $P = (y = 1|\mathbf{x}) = p(\mathbf{x})$. Variable $\mathbf{x} =$
 17 (x_1, x_2, \dots, x_k) denotes a series of independent variables. Probability $p(\mathbf{x})$ is calculated as in
 18 Eq. (1),

$$19 \quad p(\mathbf{x}_i) = \frac{\exp(\beta_0 + \beta_k x_k)}{1 + \exp(\beta_0 + \beta_k x_k)}. \quad (7)$$

20 The G-Statistic is given by

$$21 \quad G = D_{null} - D_k, \quad (8)$$

22 where term D_{null} denotes the deviance when the model is applied without independent
 23 variables, i.e., when $p(\mathbf{x}) = [1 + \exp(\beta_0)]^{-1}$. Term D_k refers to the deviance for the model
 24 with k independent variables. The difference between these two terms is often cited as a sign of
 25 goodness of fit. The greater this difference, the more important is the influence of the estimation

1 variables used. The optimal result for D is zero (Hosmer and Lemeshow, 2004). The process of
2 the proposed statistical model described above was implemented with original code developed
3 in the Matlab programming environment.

4 **4 Results and Discussion**

6 The BSTEM model was validated for the predicted erosion (m^2) after a field investigation that
7 was performed at the end of the wet period of the hydrologic year of 2013-14. The eroded area
8 at each location was successfully predicted based on the field observations at the affected area.
9 However, quantified measurements at those points were not performed, but only field
10 inspection to validate that the BSTEM results are consistent with reality. During the inspection,
11 photographs were taken at some locations where the 50 cm scaled stick was placed to highlight
12 the eroded area. However, only at the point with the most intense erosion, a close up photo was
13 taken and analyzed to quantify the erosion.

14 The evaluation of the BSTEM model results, involved the calculation of the percentiles used to
15 categorize the significance of the BSTEM calculated eroded area. The BSTEM model results
16 are in very good agreement with the behavior of the banks after the flood events. Of the twelve
17 measurement points, four were identified with no or low erosion, as the affected area was under
18 or very close to the 25th percentile and equal to $0.52 m^2$ (Table 1). In addition, the erosion sticks'
19 inspection, showed that the observed affected area at the four locations, was limited considering
20 that the bank form had not changed. The remaining eight points were identified as eroded and
21 significantly eroded based on the model results for the affected area, while the bank form had
22 changed at those locations. The affected area at the three significantly eroded locations ranged
23 from 1.399 to $2.043 m^2$, close and over the 75th percentile, which is equal to $1.38 m^2$. Location
24 (KI) (Fig. 1) presented the most significant erosion effect. The predicted eroded area was equal
25 to $2.043 m^2$ and the affected area measured at the field (and represented in the modified photo,
26 Fig. 4) was roughly $2.08 m^2$. The situation is the same for the other locations. The statistical
27 model input considers the twelve measurement locations as eroded or not eroded based on the
28 BSTEM results and the observed bank formation (Table 2).

29 The aforementioned results mean that the BSTEM outcome for the eight subsections of the first
30 campaign can be also characterized as reliable, as they are located in-between of the twelve
31 points that were successfully validated by the field inspection. The model outcome provided
32 seven subsections with potential to erosion vulnerability and one not vulnerable to erosion,

1 [based on the estimated affected area in comparison to the total area of the bank at the respective](#)
2 [river subsection](#). Therefore, they [could](#) be used as validation locations for the [assessment of the](#)
3 statistical model performance.

4 [Consequently](#), the twelve measurements of the 2nd field campaign were used to apply LR and
5 LWLR, while the eight locations of the 1st campaign were employed as validation points. The
6 first BSTEM application has provided a vulnerability assessment of the riverbank sections that
7 these eight locations assign. The riverbank areas vulnerable to erosion, and therefore the
8 associated locations are characterized as Unstable “U” and the non-vulnerable as Stable “S”.
9 Correspondingly to the LR and LWLR that deliver probabilities of erosion to occur, $P \geq 0.5$ is
10 interpreted as presence of erosion and is denoted as Unstable “U” and absence of erosion $P < 0.5$
11 as “S”, Stable. Therefore, the different statistical model forms are validated based on the erosion
12 vulnerability of the eight locations of the 1st field campaign (Tables 3 and 4).

13 The results derived from the application of the LR model, with uniform parameters for all
14 estimation points, are presented in Table 3. The values of the independent variables and the
15 BSTEM erosion estimates at the validation points are also presented in the same table. The
16 model deviance was calculated equal to 6.14 and the G-Statistic equal to 7.23. Results for the
17 erosion probability at different ungauged locations along the Koiliaris’ riverbanks obtained with
18 the LR model are presented in Fig. 5a. The values for the independent variables were [randomly](#)
19 [selected from locations among the measurement points based on](#) a 3D digital model of [River](#)
20 [Koiliaris, which was developed based on a 5 m](#) Digital Elevation Model (DEM).

21 [On the other hand](#), results for the erosion probability at the validation points derived by applying
22 LWLR with the exponential and the tri-cubic weighting functions, are presented in Table 4. The
23 graphical representation of the results for the erosion probability at the ungauged locations, is
24 provided in Figs. 5b and 5c for the exponential and tri-cubic functions respectively. In the case
25 of the exponential weighting function, the model deviance is equal to 6.27 and the G-statistic
26 equal to 5.10, while in the case of the tri-cubic function, the model deviance is equal to 5.12
27 and the G-statistic equal to 6.25.

28 Inter-comparison of estimations [of the three methods tested](#) is possible, as the x and y axis of
29 the plots ([Fig. 5](#)) are at the same scale. In addition, the validation points are shown on the plots
30 for easier [contrast](#). The [produced 3D figures \(Fig. 5\) actually work as a probability map](#)
31 [presenting](#) the probability of erosion to occur (z axis) at the specific riverbank locations when
32 a couple of independent values is met (x and y axes). In a similar work recently published

1 (Vozinaki et al., 2015), the simple LR model was applied on predicting crop damage curves
2 based on measurements of river flood depth and velocity (secondary data). The secondary data
3 required to develop the probability curves (predictions) were produced by a Monte Carlo
4 simulation in the absence of sufficient measurement data. Herein, the selected secondary values
5 [were derived](#) from the 3D river structure model [as it was previously explained](#).

6 Both LWLR models₂ involve a nonlinear parameter in the weighting function that determines
7 the correlation distance of the spatially correlated measurement points. The optimal distance in
8 each case₂ was calculated using a leave-one-out cross validation analysis₂ involving the
9 measurement locations. As a result, parameter a of the exponential weighting function₂ was set
10 to 600 m and parameter h of tri-cubic function₂ was set to 400 m.

11 The results obtained with the LR method₂ were in very close agreement with those of BSTEM
12 as the erosion presence or absence was accurately predicted at six out of the eight locations,
13 with one of the fail locations to have a narrow deviance from the set erosion presence limit.
14 Next, to improve predictions, [the LWLR method](#) was applied to account for the local spatial
15 dependence of the independent variables at the measurement locations. The LWLR model with
16 the exponential function has, overall, similar performance to the LR model. The derived results
17 are in agreement with the BSTEM estimates at seven out of the eight validation locations and
18 the approach fails at only one validation location. The application of the LWLR model with the
19 tri-cubic function leads to significant improvement of the estimates and to the accurate
20 prediction of the erosion probability at all eight validation locations. The significant result for
21 this model [was](#) the validation of a clearly unstable point (pin no. 7) which has independent
22 variables that should provide a stable indication (as delivered by LR). Another point with
23 similar characteristics (pin no. 4, [Fig. 1](#)) was correctly identified as stable. Therefore, such
24 performance is possible only when local spatial weighting functions are used.

25 The only validation point indicated as stable (pin no. 4, [Fig. 1](#)) belongs to the fourth river section
26 (between pins no. 3 and 4, Fig. 1) which as a whole₂ was determined by BSTEM as stable.
27 However, two out of the three local measurements in the same section (pins KB and KC in Fig.
28 1)₂ showed signs of erosion after the inspection. Generally though, apart from limited locations,
29 the banks of that section did not show erosion signs due to the presence of dense seasonal
30 riparian vegetation. The erosion probability estimation at this point is affected significantly, at
31 local scale, by the spatially correlated measurement points with low vulnerability to erosion.
32 Similarly, validation points 6 and 7 are also affected by the close presence of measurement

1 locations with low vulnerability to erosion. This explains the difficulty in predicting erosion at
2 these points. The model results may confirm the presence or absence of erosion at the validation
3 points, but they are quite different from the targeted values of zero for no erosion and one for
4 erosion presence. This is expected to improve when a larger dataset with greater variability of
5 the independent variables effect on erosion becomes available.

6 The graphical representation of the LWLR model results at the discretized river section (Figs.
7 5b and 5c) shows a significant difference in performance for the two weighting functions. The
8 tri-cubic function (Fig. 5c) delivers more reliable results as it is clearly considers the variability
9 of the independent variables inside the correlation distance. This can be observed [through](#) the
10 color variability in the graph of Fig. 5c that represents the variability of the erosion occurrence
11 probability. On the other hand, the exponential function (Fig. 5b) shows a smooth change in
12 probability for the different pairs of independent variable values. This can be explained in terms
13 of the function shape behavior and the correlation distance. The tri-cubic function is herein
14 applied in a shorter correlation distance according to the cross validation results, which can
15 capture the local dependence of the explanatory variables that in longer distances are smoothed
16 due to the presence of more data.

17 The LWLR method with the tri-cubic function, yields the highest value for the G-Statistic for
18 the selected independent variables. Therefore, it can be viewed as the optimum approach to
19 calculate the erosion presence probability at local scale. The G-Statistic can be also used to
20 assess the impact and importance of each independent variable on the estimates. Each variable
21 was separately applied both in LR and LWLR. The G-Statistic obtained its highest values when
22 the cross section width was applied. The results of the statistical term, improved by 12% and
23 20%, respectively, compared to the bank slope application.

24 [The LR based models results suggest that riverbank erosion probability generally increases as](#)
25 [the bank slope increases and the river cross section decreases. This is due to an increase of the](#)
26 [flow velocity that removes the non-cohesive soil components from the banks. Based on field](#)
27 [measurements analysis, the bank material at the Koiliaris River was classified as “fine rounded](#)
28 [sand”. The fine rounded material is easier removed due to its low resistance and increased flow](#)
29 [friction. This characteristic is associated with the LR based models results, as they provide](#)
30 [mainly favorable probabilities of riverbank erosion at the validation points. However, to connect](#)
31 [the soil properties effect with the probability of erosion that results from geomorphological](#)
32 [variables in detail, the LR based models should account also for soil properties, such as particle](#)

1 [size distribution and the bulk density that consider also mechanical properties of the riverbanks.](#)
2 [This is a task that the authors plan to address at a near future research.](#)

3 The proposed statistical model is a useful, fast, efficient and fairly easy to apply tool that
4 requires information from easy to determine geomorphological and/or hydrological variables.
5 This tool provides a quantified measure of the erosion probability along the riverbanks, and
6 could be used to assist managing erosion and flooding events. [On the other hand,](#) BSTEM model
7 [can be successfully applied to determine the potential riverbank eroded area \(\$L^2\$ \).](#) Both are
8 useful, depending on data and software availability, in providing information regarding the
9 vulnerability of riverbanks to erosion.

11 **5 Conclusions**

12 The BSTEM model set up provides reliable results regarding the potential erosion vulnerability
13 of the riverbanks that can be used to validate the estimations of the proposed statistical model.
14 On the other hand, the proposed LR based statistical model, estimates efficiently the erosion
15 probability at the riverbanks, using two secondary variables that affect significantly the
16 presence or absence of erosion. However, in LWLR, locality is [of utmost importance](#); the
17 location of the new couple of secondary variables was used to identify and weight the effect of
18 spatially correlated measurement points in order to calculate the model parameters. The
19 proposed methodology, LWLR, exploits the local information of independent variables and
20 translates it successfully to bank erosion probability. This is not a typical regression estimation
21 based on global parameters but herein, the model parameters are calculated iteratively for the
22 new couples of secondary variables.

23 The LR method performs satisfactorily in the plain form where uniform parameters are
24 considered for all estimation points. Difference from the BSTEM results is observed only at
25 two of the eight validation points. The LWLR method with the exponential weighting function
26 gives results similar to those of LR. The LWLR method with the tri-cubic function provides
27 significantly improved estimates which coincide with the BSTEM results at all validation
28 points. The graphical presentation of the results in the discretized river section shows that the
29 erosion probability increases with bank slope and decreases with cross section width. This is
30 also confirmed by the positive sign of the bank slope coefficients and the negative sign of the
31 cross section width coefficients in all LR applications. The deviance and the G-Statistic results

1 show that the cross section width parameter is more important than bank slope for the estimation
2 of erosion probability at the banks of the Koiliaris River.
3 This work presents the framework of a methodology that can be applied in order to estimate the
4 probability of erosion at specific riverbank locations considering explanatory and easy to
5 determine secondary variables. Channel geomorphological characteristics, such as cross section
6 and bank slope, are relatively easy to be determined at unmeasured locations by using a digital
7 elevation model. On the other hand, hydrological variables or bank material requires extensive
8 field measurements in order characteristic variables to be considered as secondary information.
9 Such measurements did not take place during the field campaigns as it was not in the context
10 of this work. The developed statistical tool provides an alternative proposition for the estimation
11 of riverbank locations vulnerable to erosion, which requires limited information on explanatory
12 variables, yet can provide vulnerable location estimates with increased reliability. It is therefore
13 considered as a very promising approach for the estimation of riverbank erosion probability.
14 The tool is proposed as a supplementary solution to the riverbank erosion identification issue.

15

16 **Author contribution**

17 E. A. Varouchakis developed the statistical model, the model code and performed the
18 simulations. Along with G. V. Giannakis, M. A. Lilli and N. P. Nikolaidis they designed and
19 carried out the field campaigns, while with the aid of G. P. Karatzas, they analyzed the collected
20 data and the model results. E. Ioannidou performed part of the model simulations. M. A. Lilli
21 and N. P. Nikolaidis applied the BSTEM model. Finally, E. A. Varouchakis prepared the
22 manuscript with the contribution of all co-authors.

23

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3

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1 Table 1. Amount of bank erosion at the measurement locations (Fig. 1) - Modelling
 2 results obtained by BSTEM.

Eroded area (m ²)				
Map location	1 st flood	2 nd flood	3 rd flood	Cumulative effect
KA	0.440	0.404	0.349	1.193
KB	0.566	0.510	0.394	1.470
KC	0.498	0.512	0.389	1.399
KD	0.411	0.410	0.328	1.149
KE	0.459	0.437	0.320	1.216
KG	0.258	0.255	0.213	0.726
KZ	0	0	0	0
KH	0.207	0.187	0.145	0.539
KJ	0.368	0.421	0.357	1.146
KI	0.741	0.728	0.574	2.043
KK	0	0	0	0
KL	0.167	0.162	0.132	0.461

3

1 Table 2. Presence (1) or absence (0) of erosion at measurement locations using a binary
 2 indication for the statistical model (LR and LWLR) set up based on inspection and
 3 BSTEM results. Columns 3 and 4 present the measured independent geomorphological
 4 variables.

Map location	Presence/Absence of erosion	Bank slope (degrees)	Cross section width (m)
KA	1	60	9.00
KB	1	75	9.25
KC	1	65	8.75
KD	1	55	9.00
KE	1	85	10.76
KG	1	60	11.55
KZ	0	75	10.00
KH	0	65	13.5
KJ	1	60	13.35
KI	1	75	7.60
KK	0	70	13.00
KL	0	50	9.00

5

1 Table 3. Result of LR application at the eight validation locations (Fig. 1). The
 2 independent variables used and the BSTEM estimates are also presented. In column 4,
 3 S denotes stable and U unstable bank locations.

Validation points	Bank slope (degrees)	Cross section width (m)	BSTEM erosion estimates	Erosion (P) LR
1	84	9.25	U	0.84
2	58	9.05	U	0.64
3	81	9.35	U	0.82
4	33	9.00	S	0.38
5	82.5	8.75	U	0.85
6	44	9.00	U	0.49
7	27	9.26	U	0.30
8	57	9.25	U	0.62

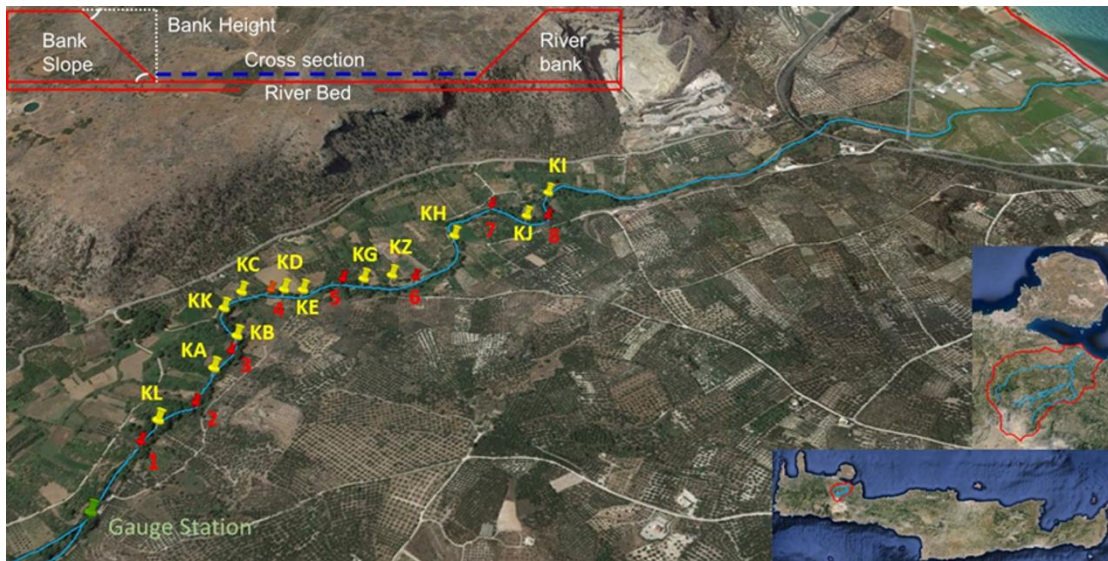
4

1 Table 4. Result of LWLR application at the eight validation locations (Fig. 1). The LR
 2 estimates, the independent variables used and the BSTEM estimates are also presented.
 3 With bold face the diverged values are indicated. In column 4, S denotes stable and U
 4 unstable bank locations.

Validation point	Bank slope (degrees)	Cross section width (m)	BSTEM Erosion estimates	Erosion (P) LR	Erosion (P) LWLR (exp. model)	Erosion (P) LWLR (tri-cubic model)
1	84	9.25	U	0.84	0.85	0.86
2	58	9.05	U	0.64	0.58	0.74
3	81	9.35	U	0.82	0.75	0.87
4	33	9.00	S	0.38	0.27	0.25
5	82.5	8.75	U	0.85	0.81	0.82
6	44	9.00	U	0.49	0.54	0.53
7	27	9.26	U	0.30	0.21	0.52
8	57	9.25	U	0.62	0.64	0.73

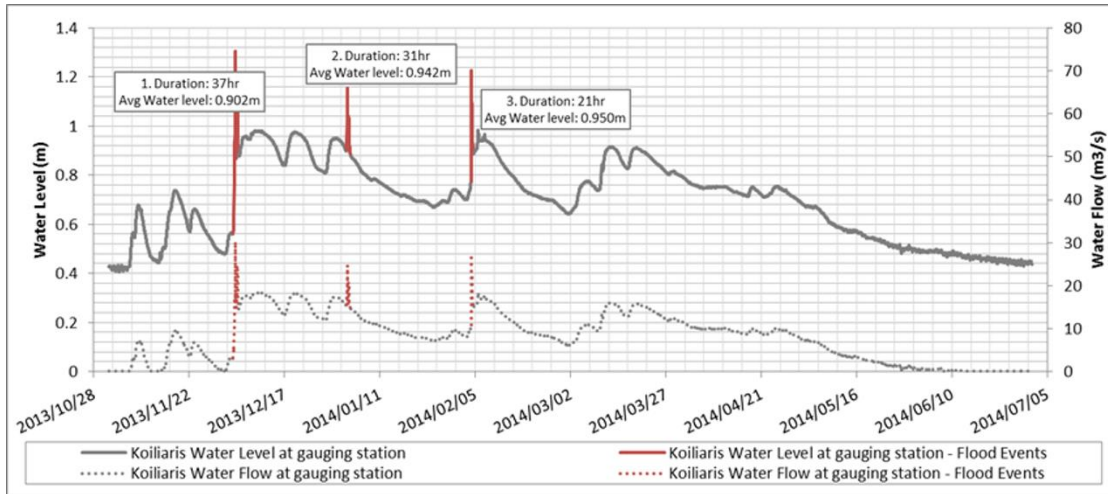
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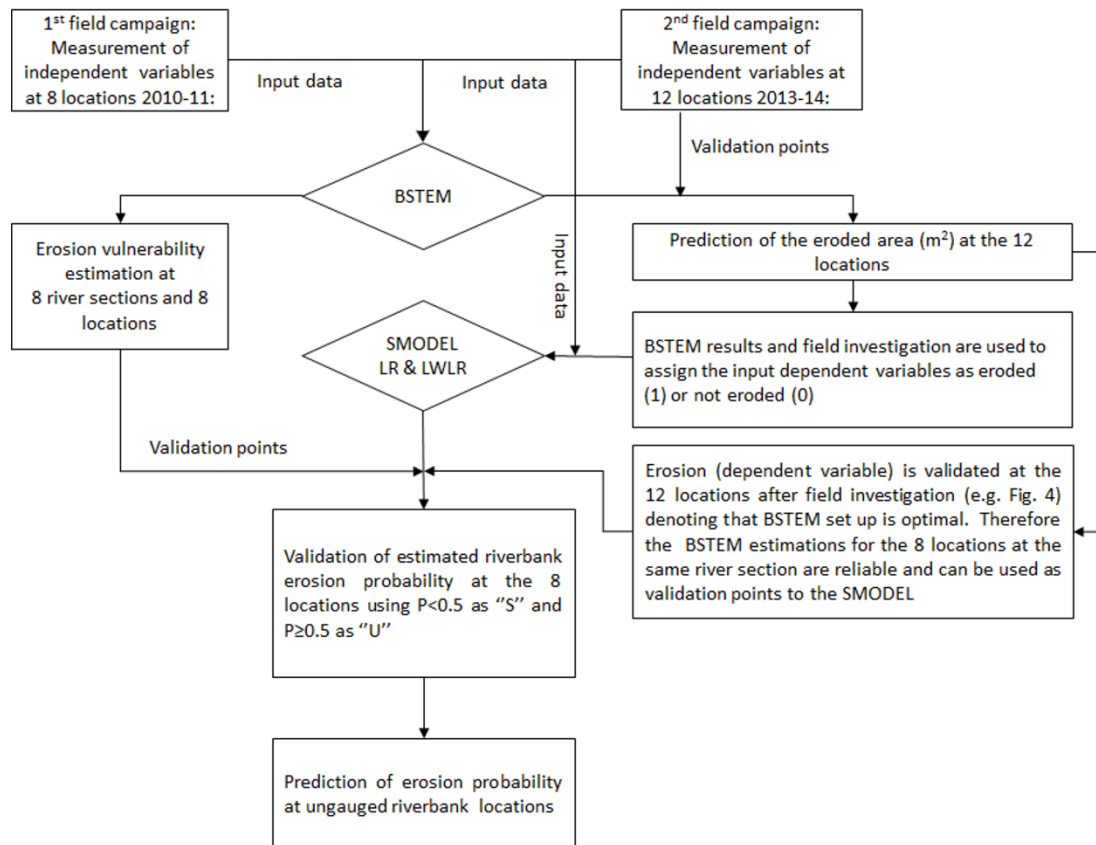
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3 Figure 1. The downstream part of the Koiliaris River located in the western part of the
4 island of Crete. The yellow pins represent the measurement locations, the red pins the
5 validation locations and the green pin the Gauge Station located at the intersection of
6 the the Koiliaris River with the Keramianos tributary. A representation of the measured
7 geomorphological values is provided in the upper left corner.



1

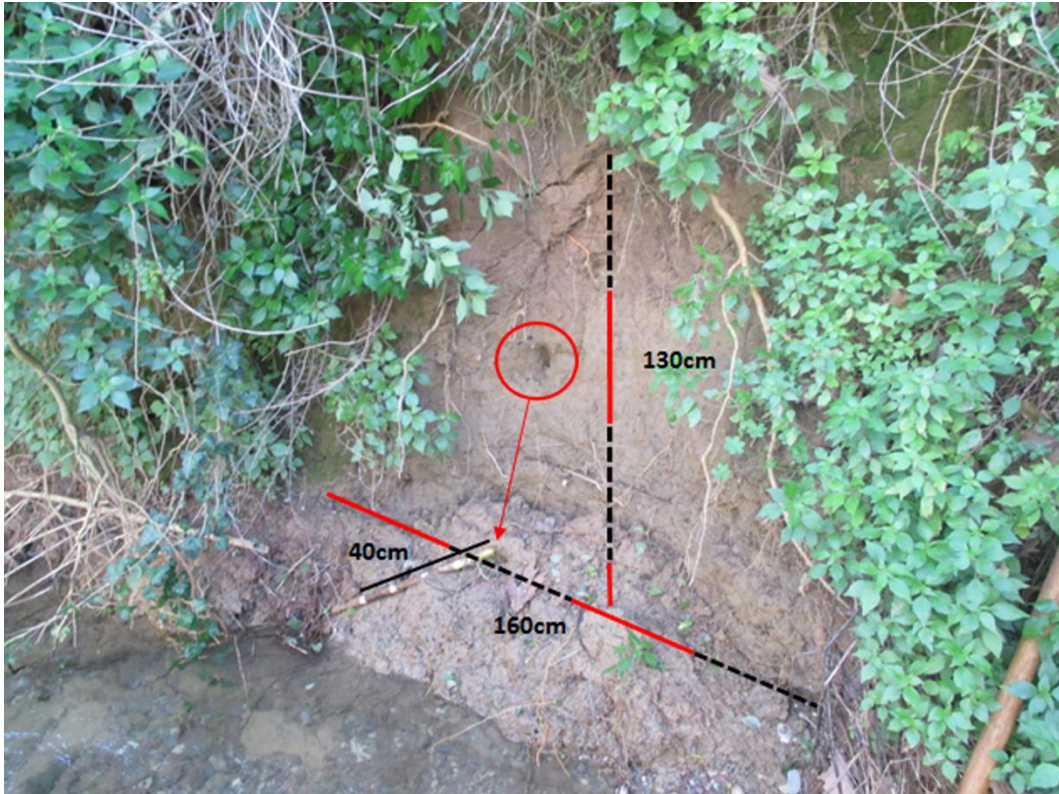
2 Figure 2. Typical hydrograph of the Koiliaris River at the Gauge Station (November
 3 2013 – June 2014).



1

2 Figure 3. Process flowchart that presents the combined application of the BSTEM and
 3 of the proposed statistical model (SMODEL) based on LR principles. The notation ‘‘S’’
 4 and ‘‘U’’ correspond to Stable and Unstable riverbanks respectively.

5

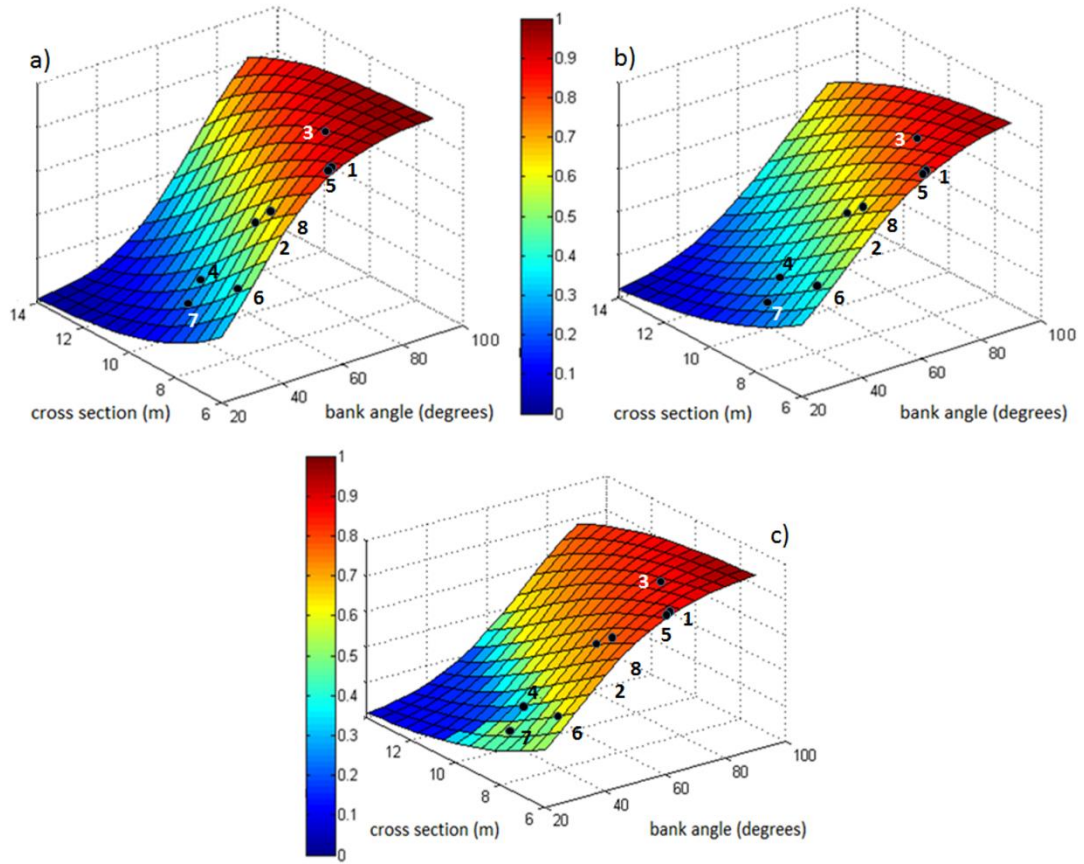


1

2 Figure 4. Photo highlight of the riverbank location (KI) with the most intense observed
3 erosion accompanied by the appropriate scaled tools to provide a rough estimate of the
4 eroded area.

5

1



2

3 Figure 5. Erosion probability predictions using a) LR, b) LWLR with the exponential
4 weighting function and c) LWLR with the tri-cubic weighting function versus variable
5 independent values at random ungauged Koiliaris' riverbank locations. The black dots
6 indicate the eight validation points.