Equations used in RillGrow

Infiltration is calculated using the explicit Green-Ampt Model of Salvucci and Entekhabi (1994):

$$f(t)_{\{SE\}} = K_s \left\{ \frac{\sqrt{2}}{2} \tau^{\frac{-1}{2}} \frac{+2}{3} - \frac{\sqrt{2}}{6} \tau^{\frac{1}{2}} + \frac{1-\sqrt{2}}{3} \tau \right\} \qquad \text{eq. S1}$$

$$\chi = \frac{\eta(H+\psi f)}{K_S} = \frac{S^2}{2K_S^2} \qquad \text{eq. S2}$$

$$\tau = \frac{t}{t+\chi} \qquad \text{eq. S3}$$

$f(t)_{\{SE\}}$ is the rate of infiltration
χ is time
τ is a dimensionless model variable.

Reference
Salvucci, G. D. and Entekhabi, D. (1994). Explicit Expressions for Green-Ampt (Delta Function Diffusivity) Infiltration and Cumulative Storage, Water Resour Res, 30, 2661–2663, doi: 10.1029/94WR01494.

The *speed of overland flow* is calculated using the Darcy-Weisbach equation:

$$Fs = \sqrt{\frac{K \cdot G \cdot Hr \cdot Ts}{ff}} \qquad \text{eq. S4}$$

Fs = flow speed
K = constant
G = gravitational acceleration
Hr = hydraulic radius
Ts = top slope i.e. slope of cell-cell water surface
ff = friction factor, here calculated as:

$$ff = A \cdot Re^B \qquad \text{eq. S5}$$

A = constant
B = constant
Re = Reynolds' number

Splash redistribution on a single cell is based on equation (20) in Planchon et al. (2000):

$$dZ = K \cdot KE \cdot \nabla^2 z \qquad \text{eq. S6}$$

dZ = change in elevation
K = constant
KE = raindrop kinetic energy
$\nabla^2 z$ = Laplacian

$$\nabla^2 z = \frac{\left(z_{i+1,j} + z_{i-1,j} + z_{i,j+1} + z_{i,j-1} - 4z_{i,j}\right)}{p^2} \qquad \text{eq. S7}$$

$z_{i,j}$ = elevation of cell i,j

p = cell size

Reference

Planchon, O., Esteves, M., Silvera, N., and Lapetite, J. M. (2000). Raindrop erosion of tillage induced microrelief: possible use of the diffusion equation, Soil Tillage Res, 15–16, 1–14.

Transport capacity is calculated using equation (5) in Nearing et al. (1997):

$$log(q_s) = \alpha + \beta \frac{e^{\gamma+\delta log\,(\varpi)}}{1+e^{\gamma+\delta log\,(\varpi)}} \qquad \text{eq. S8}$$

q_s = unit sediment load
ω = stream power
α, β, γ, δ = constants

Reference

Nearing, M.A., Norton, L.D., Bulgakov, D.A., Larionov, G.A., West, L.T. and Dontsova, K. (1997). Hydraulics and erosion in eroding rills. Water Resources Research 33(4), 865-876

Flow detachment is calculated using equation (10) in Nearing (1991):

$$e = K \cdot C \cdot P \cdot h^{0.5} \cdot S^{1.5} \qquad \text{eq. S9}$$

e = detachment rate per unit area
K = constant
C = Chezy coefficient:

$$C = \left(8 \cdot \frac{g}{f}\right)^{0.5} \qquad \text{eq. S10}$$

g = gravitational constant
f = Darcy-Weisbach friction factor

$$f = \frac{(8 \cdot g \cdot h \cdot S)}{u^2} \qquad \text{eq. S11}$$

h = water depth
S = slope
u = flow speed

$$P = \frac{1}{2} - \frac{\Psi(T-\tau_b)}{s_T^2 + s_\tau^2} \qquad \text{eq. S12}$$

Ψ = cumulative probability function of a standard normal deviate (here calculated using an approximation from Abramowitz & Stegun, 1965)
T, S_T, S_τ = constants
τ_b = constant · τ

$$\tau = \rho \cdot g \cdot h \cdot S \qquad \text{eq. S13}$$

ρ = density of water

References

Abramowitz, M. and Stegun, I.A. (Eds.) (1965). Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables. Dover Publications Inc., New York, 1046 p.
Nearing, M.A. (1991). A probabilistic model of soil detachment by shallow turbulent flow. Transactions of the American Society of Agricultural Engineers 34(1), 81-85.

Deposition is given by equation 12 in Lei et al. (1998):

$$D = \frac{(SL - TC)}{(SL + TC)} \quad \text{eq. S14}$$

where:
D = deposition
SL = sediment load
TC = transport capacity

Reference

Lei, T., M. A. Nearing, K. Haghighi, and V. F. Bralts (1998). Rill erosion and morphological evolution: A simulation model, Water Resour. Res., 34(11), 3157–3168, doi:10.1029/98WR02162.

Table S1: Range of RillGrow model parameters for sampling with the LHC approach

Field experiment	Min	Max
Constant n for splash efficiency [$sec^2\ kg^{-1}m^{-1}$]	5.00E+10	2.50E+13
Base level e.g. distance below lowest DEM point to flume lip [cm]	5	50
Maximum flow speed [$mm\ sec^{-1}$]	500	1100
Constant k for detachment [$kg\ m^{-3}$]	0.3	0.9
Radius of soil shear stress 'patch' [mm]	20	120
When saturated, threshold shear stress for slumping [$kg\ m\ s^2$]	0.1	5
Angle of rest for saturated slumped sediment [%]	10	110
Laboratory experiment	**Min**	**Max**
Constant n for splash efficiency [$sec^2\ kg^{-1}m^{-1}$]	5.00E+10	2.50E+13
Base level e.g. distance below lowest DEM point to flume lip [cm]	5	50
Maximum flow speed [$mm\ sec^{-1}$]	500	1200
Constant k for detachment [$kg\ m^{-3}$]	0.3	0.9
Radius of soil shear stress 'patch' [mm]	20	120
When saturated, threshold shear stress for slumping [$kg\ m\ s^2$]	0.1	5
Angle of rest for saturated slumped sediment [%]	10	110

Figure S1: Range of parameters of best 30 models for different combinations objective functions at the field experiment. Note that y-axis starts at 0.3. The column colours correspond to the consideration of EC, SY or SY and EC. The objective functions and their abbreviations are explained in table 2 in the manuscript.

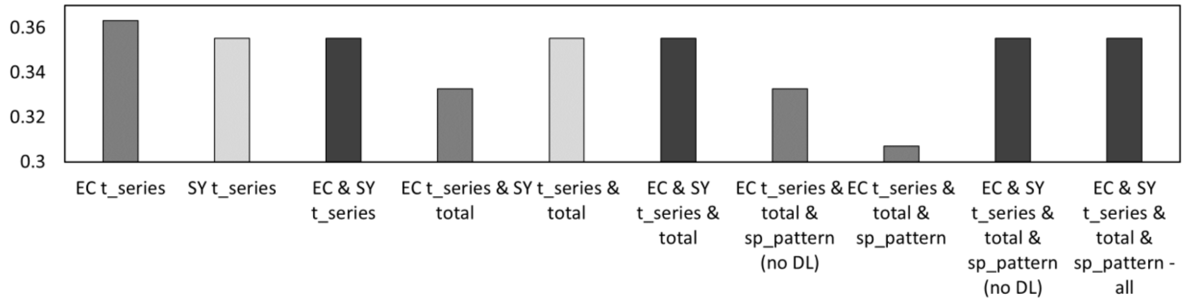

Figure S2: Range of parameters of best 30 models for different combinations objective functions at the laboratory experiment. Note that the y-axis values start at 0.2. The objective functions and their abbreviations are explained in table 2 in the manuscript.

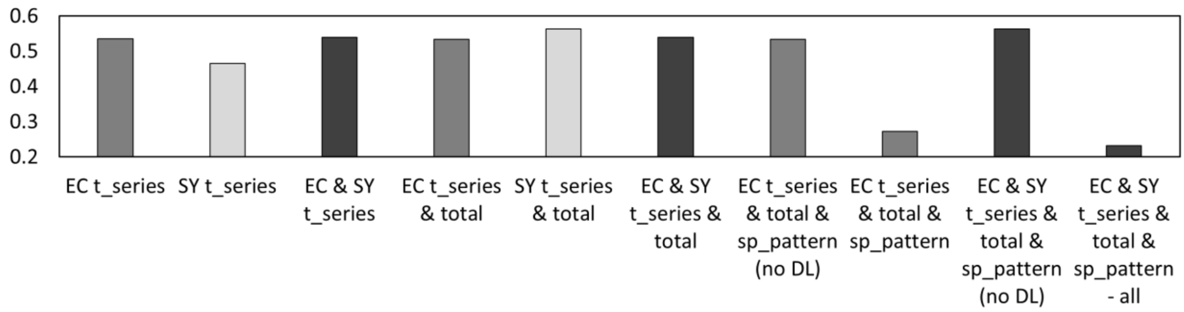

Figure S3: Scatter matrix of metrices according to different objective functions for the field experiment before filtering for unplausible models (EC in m, except sim DL that has no unit; SY in kg).

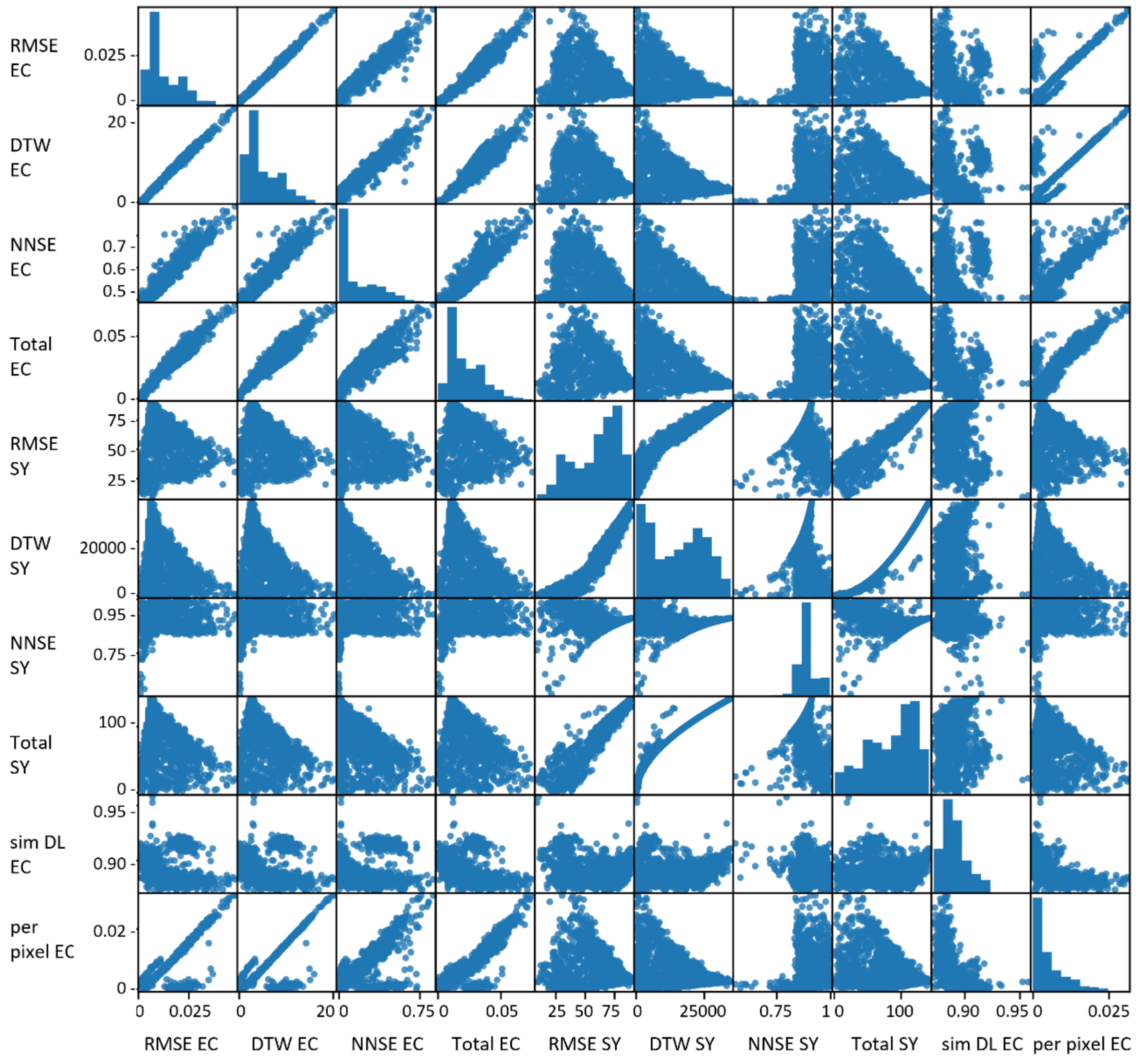

Figure S4: Scatter plot for the field experiment for the model parameter DEM base level and different metrics of the objective functions.

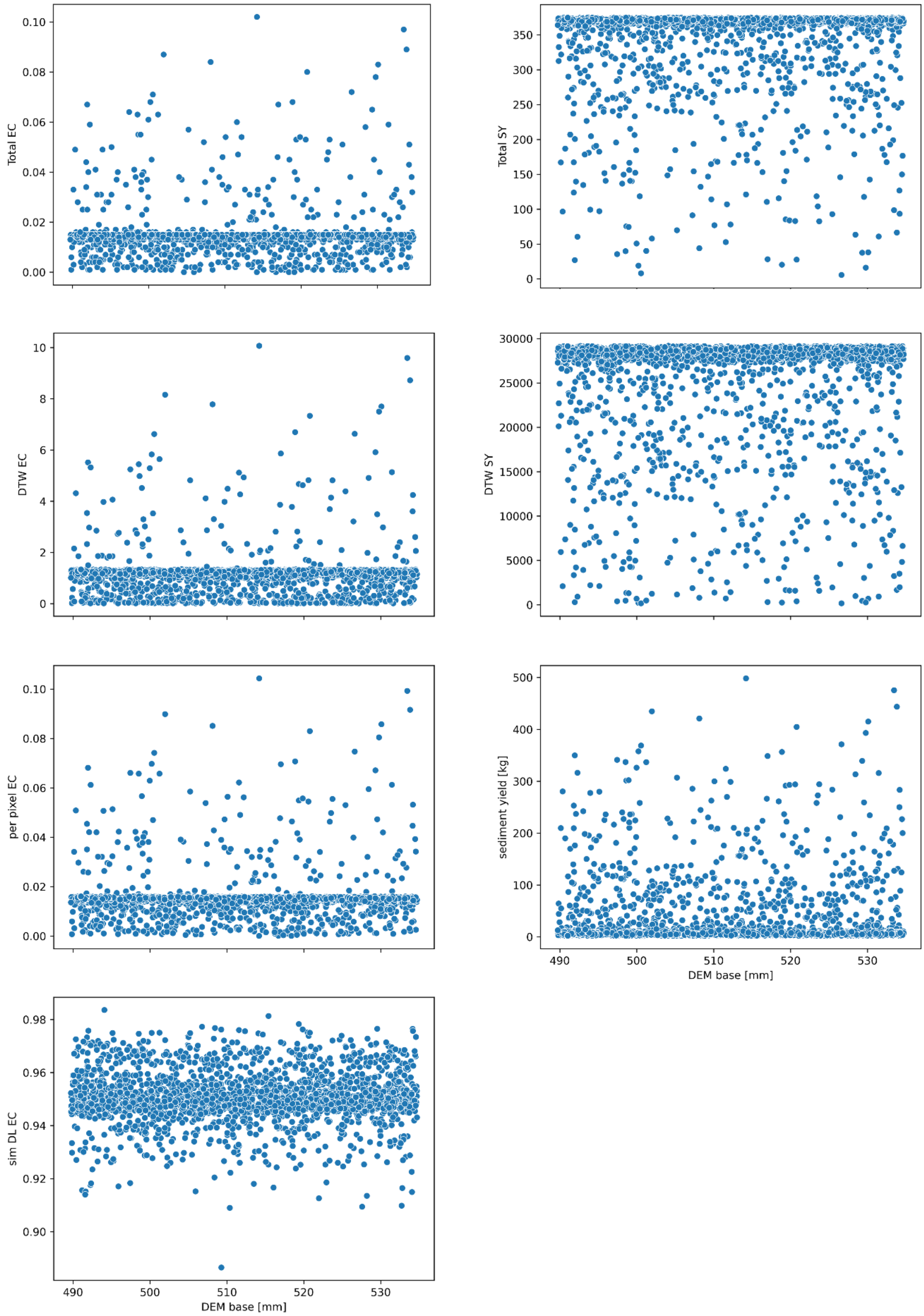

Figure S5: Scatter plot for the field experiment for the model parameter splash efficiency and different metrices of the objective functions.

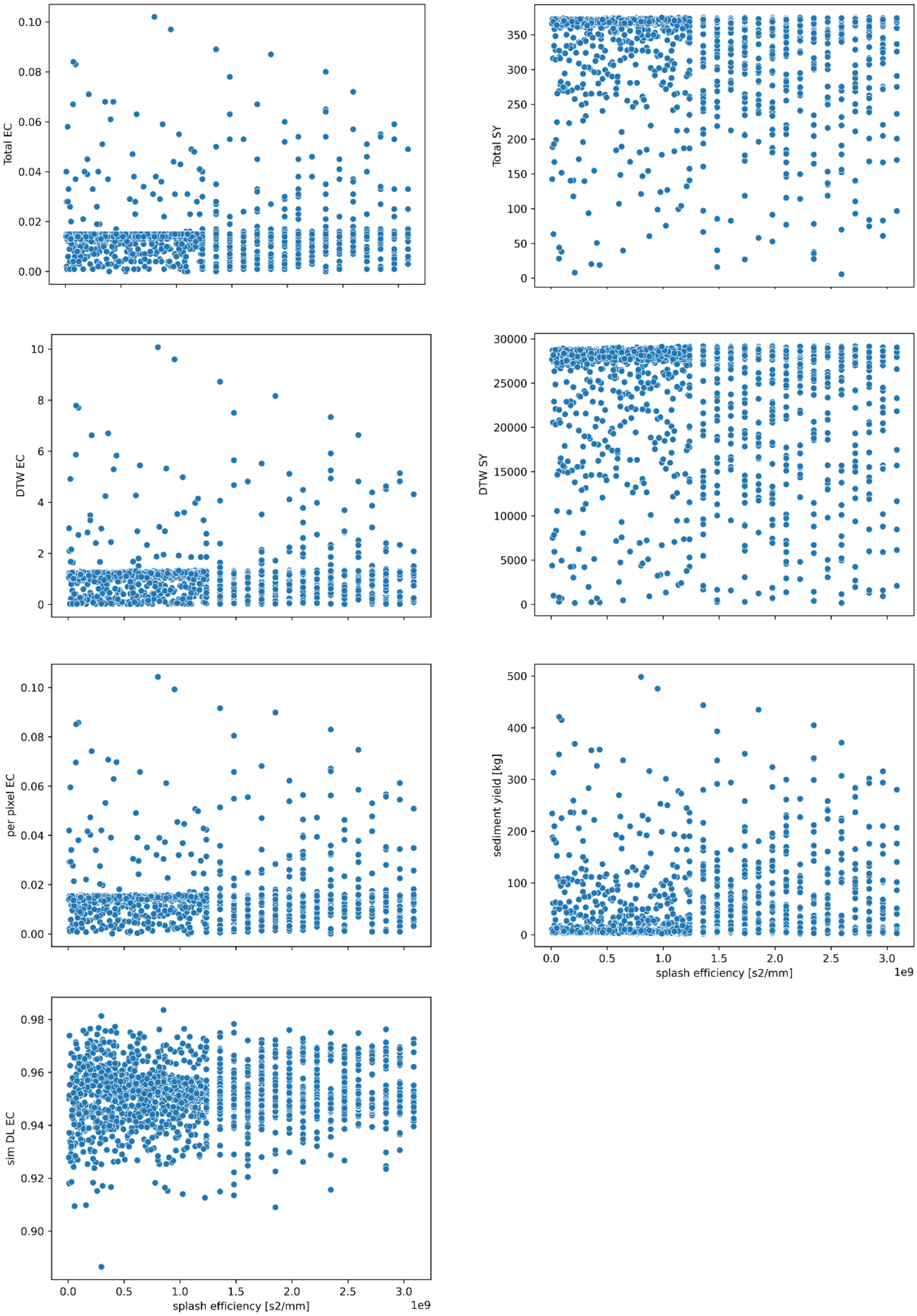

Figure S6: Scatter plot for the field experiment for the model parameter maximum flow velocity and different metrics of the objective functions.

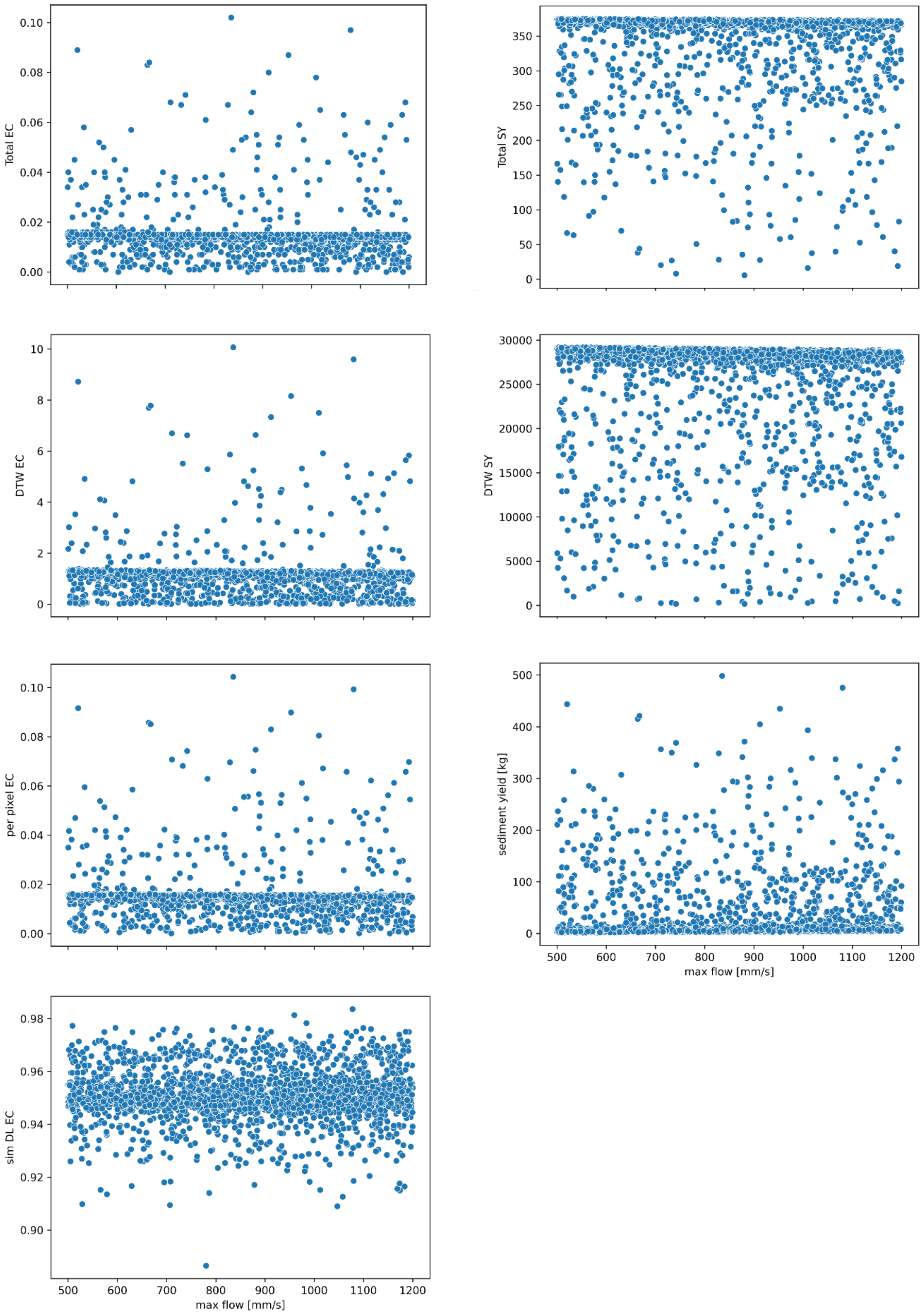

Figure S7: Scatter plot for the field experiment for the model parameter flow detachment and different metrices of the objective functions.

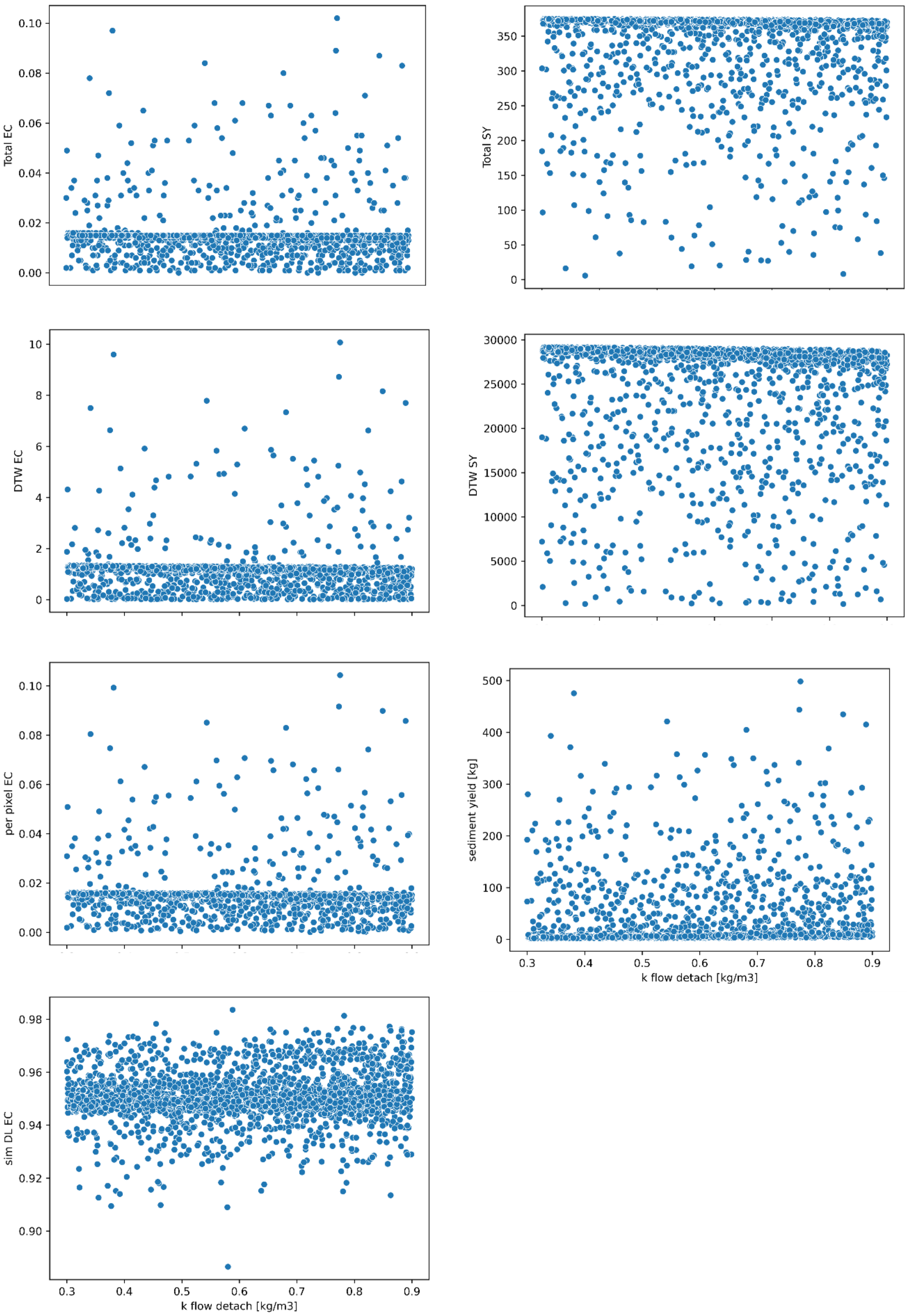

Figure S8: Scatter plot for the field experiment for the model parameter radius of soil shear stress and different metrices of the objective functions.

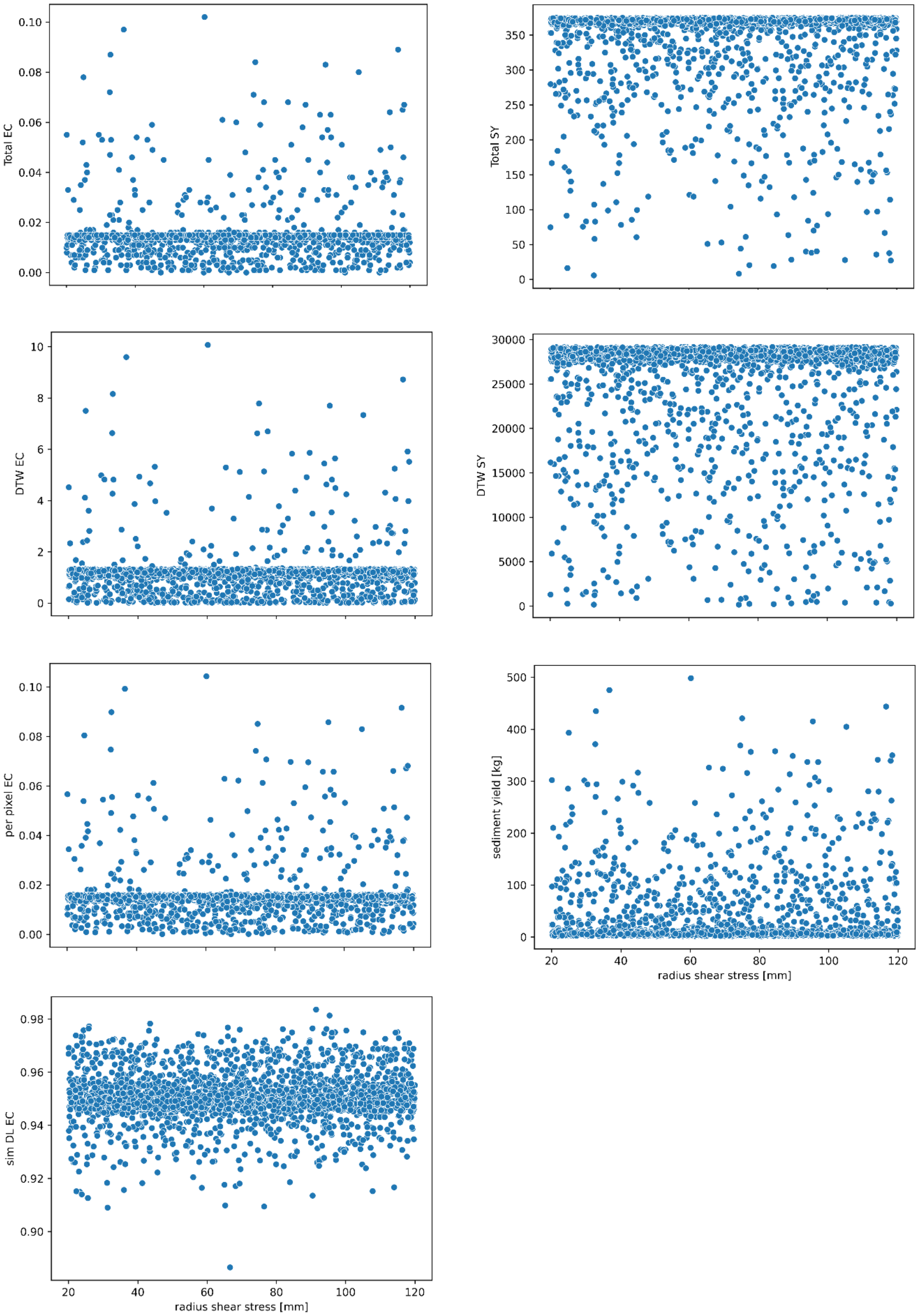

Figure S9: Scatter plot for the field experiment for the model parameter threshold shear stress for slumping and different metrices of the objective functions.

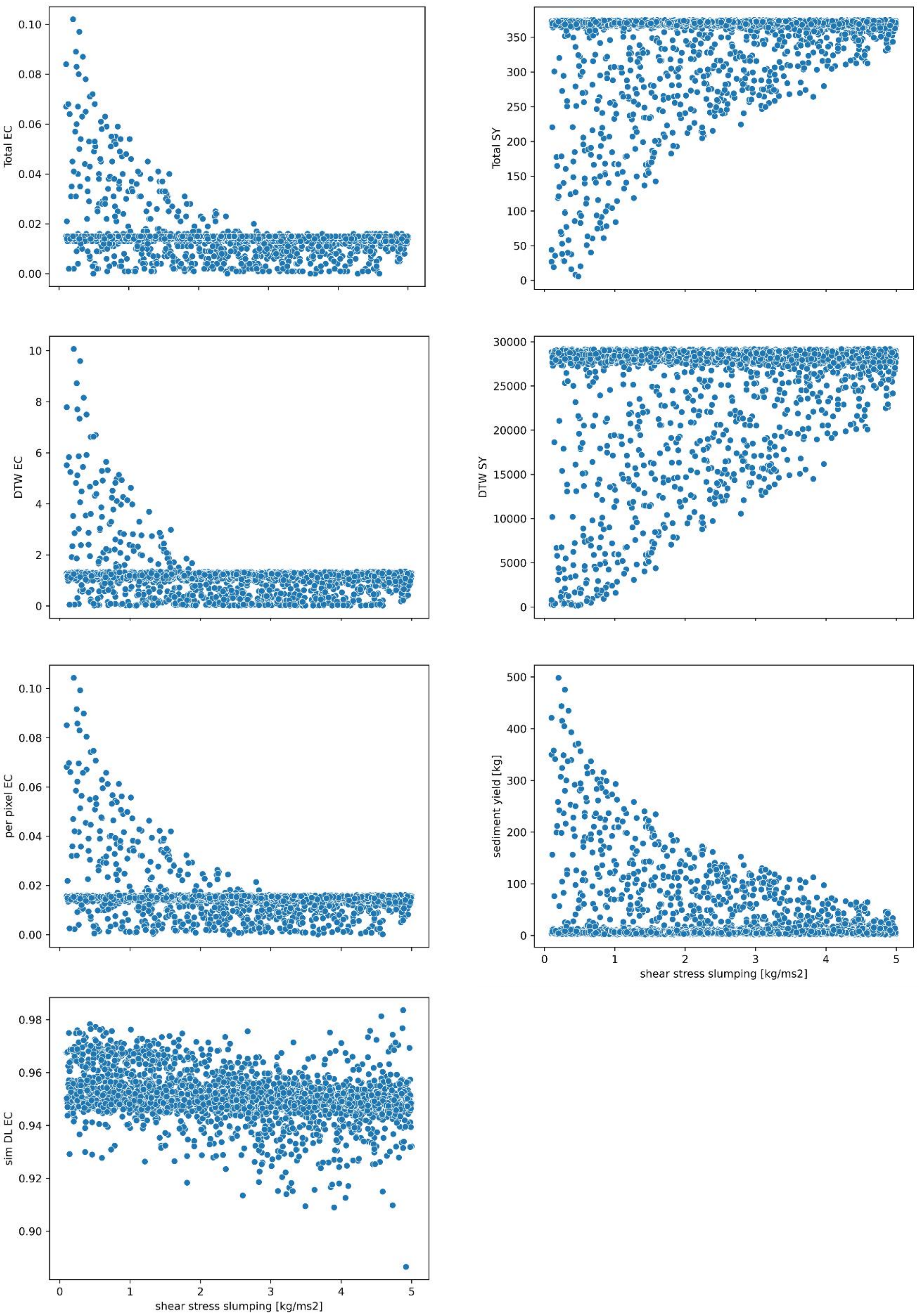

Figure S10: Scatter plot for the field experiment for the model parameter angle of rest for slumped soil and different metrices of the objective functions.

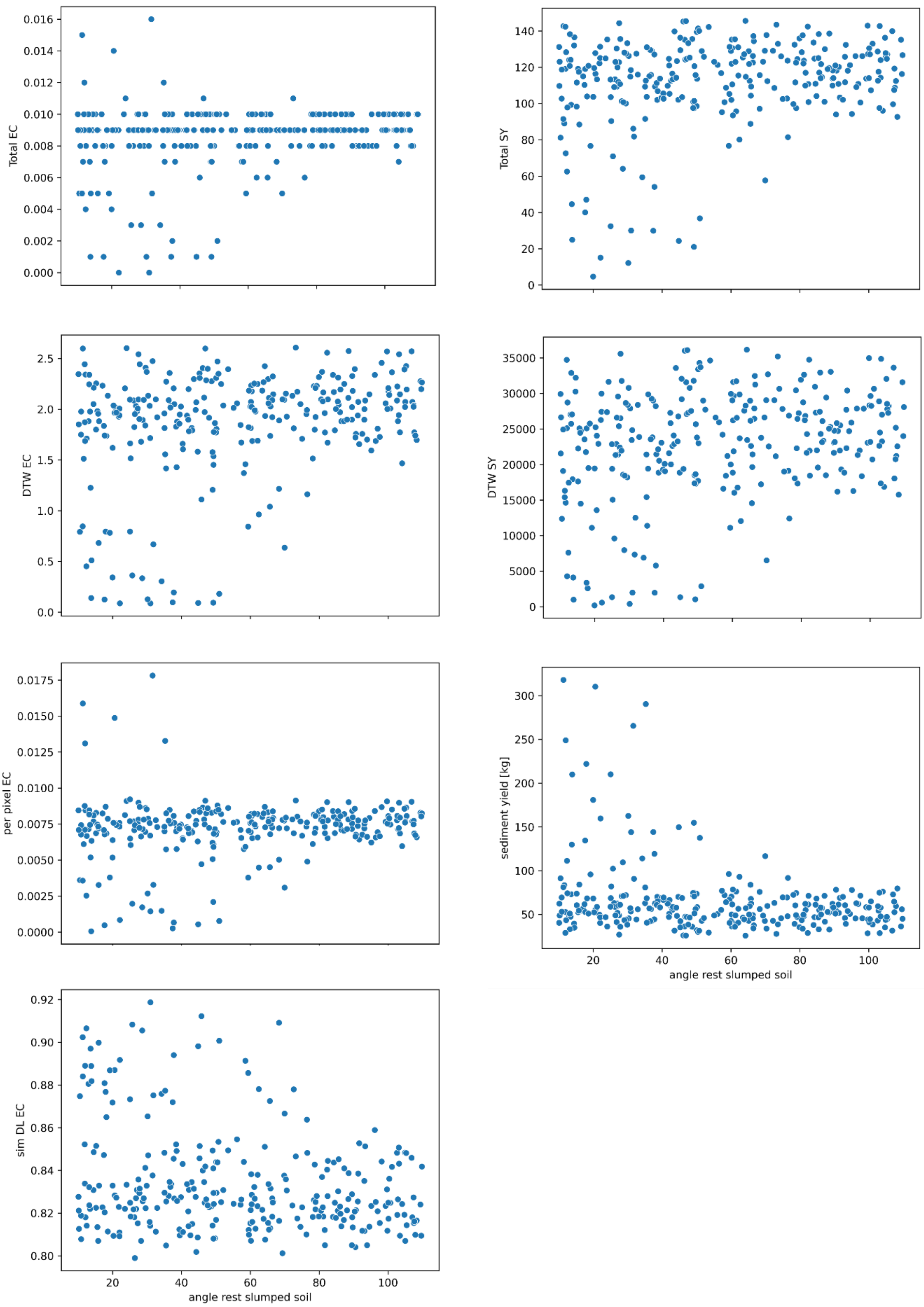

Figure S11: Scatter plot for the laboratory experiment for the model parameter DEM base level and different metrics of the objective functions.

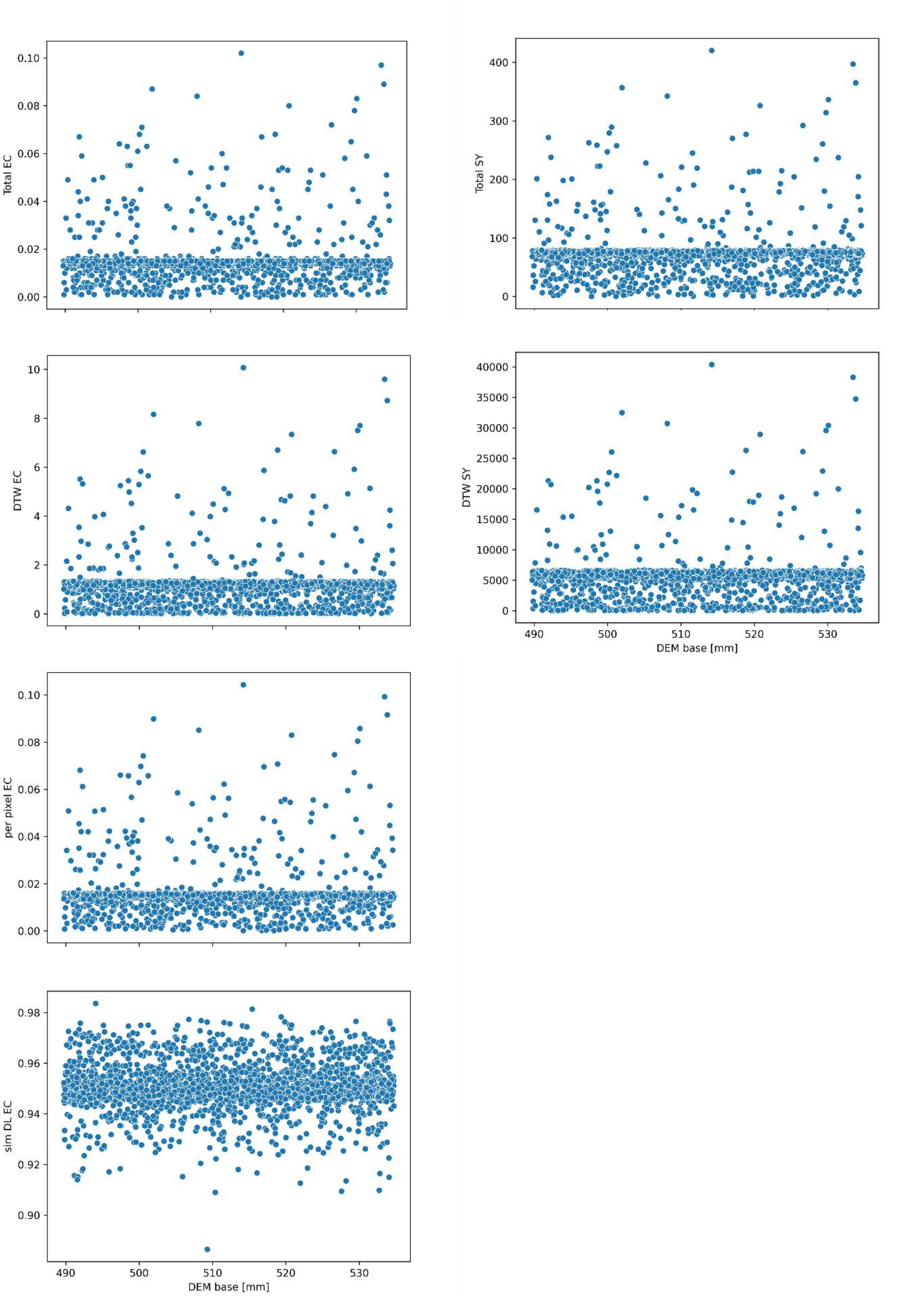

Figure S12: Scatter plot for the laboratory experiment for the model parameter splash efficiency and different metrics of the objective functions.

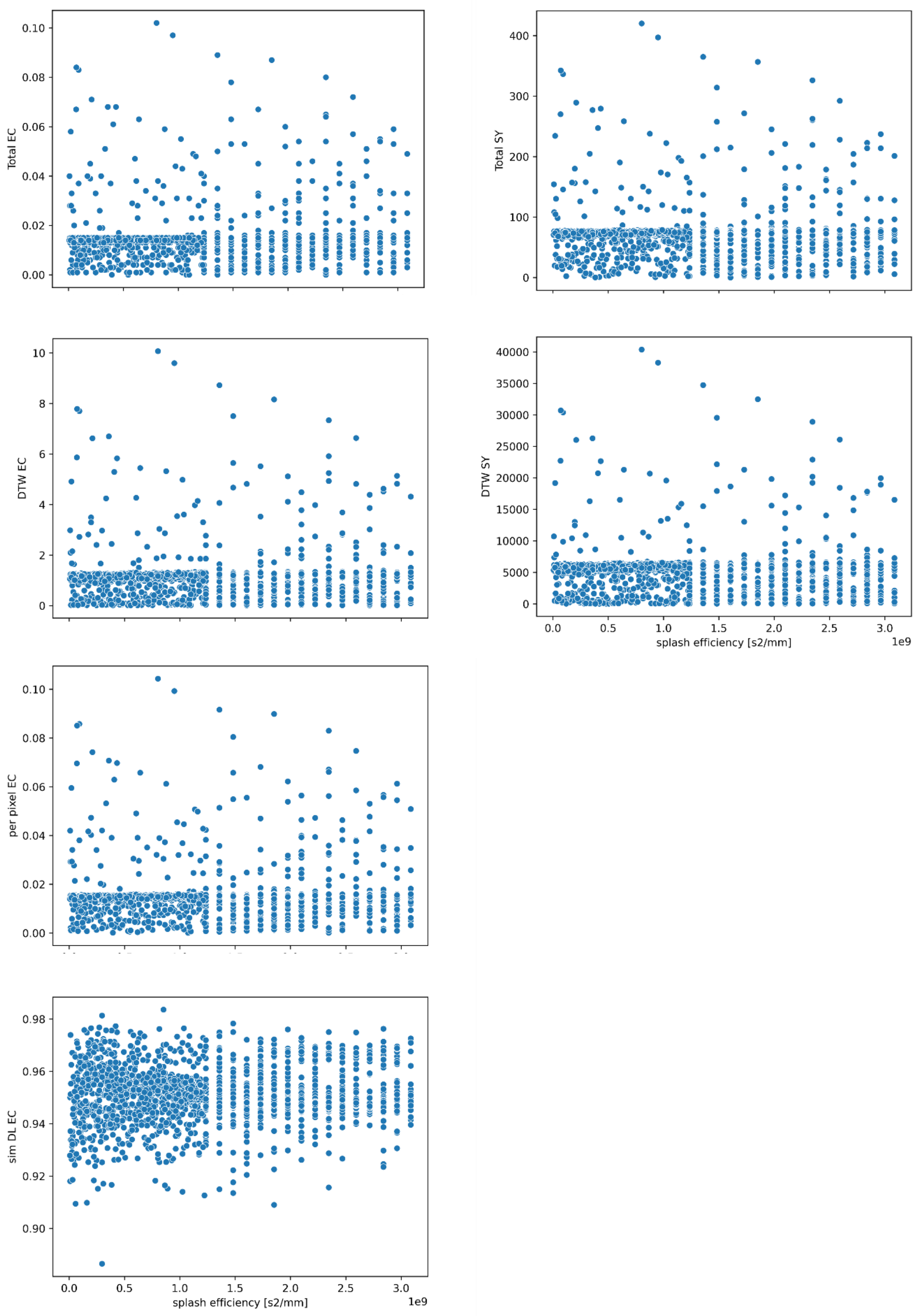

Figure S13: Scatter plot for the laboratory experiment for the model parameter maximum flow velocity and different metrices of the objective functions.

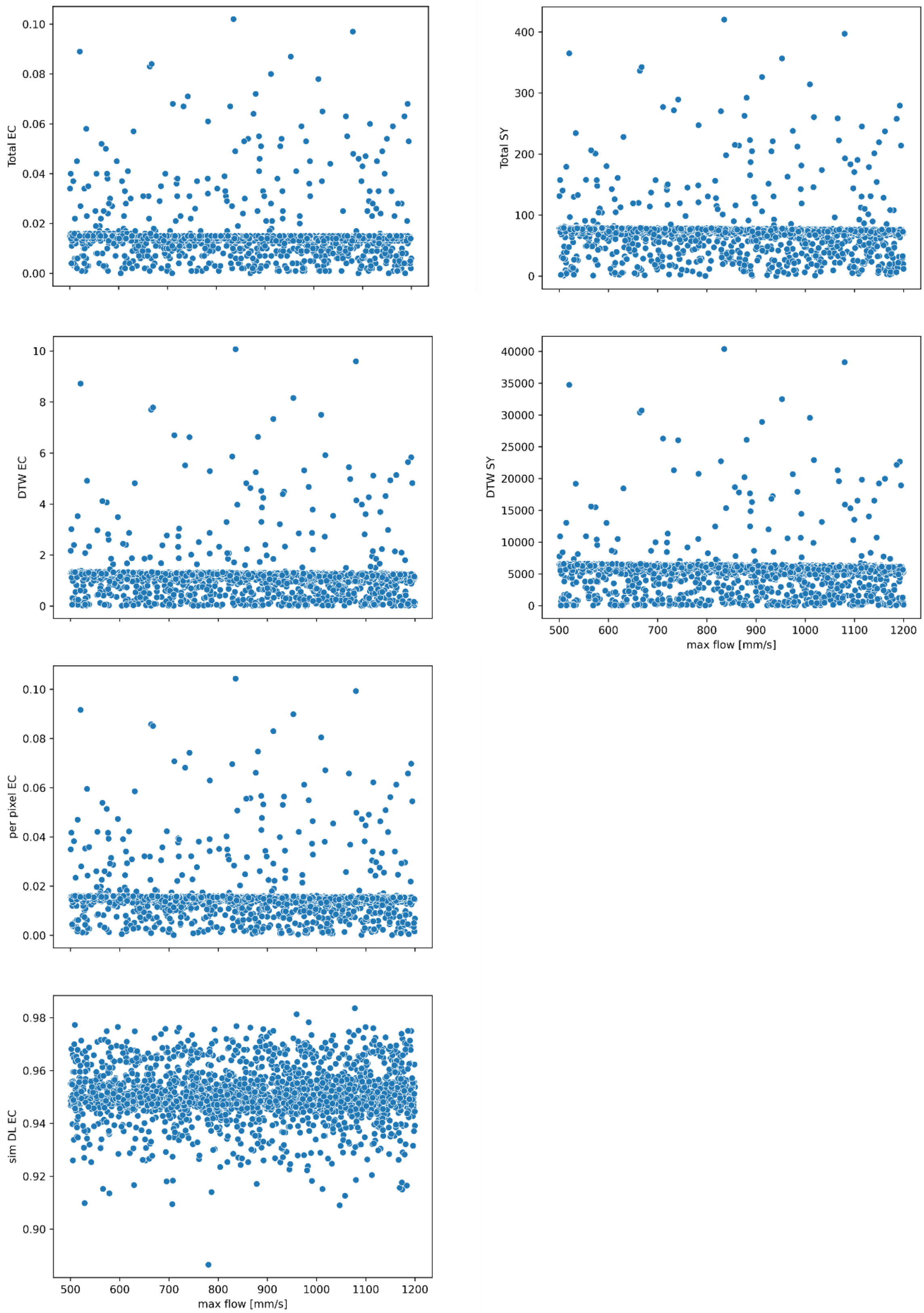

Figure S14: Scatter plot for the laboratory experiment for the model parameter flow detachment and different metrices of the objective functions.

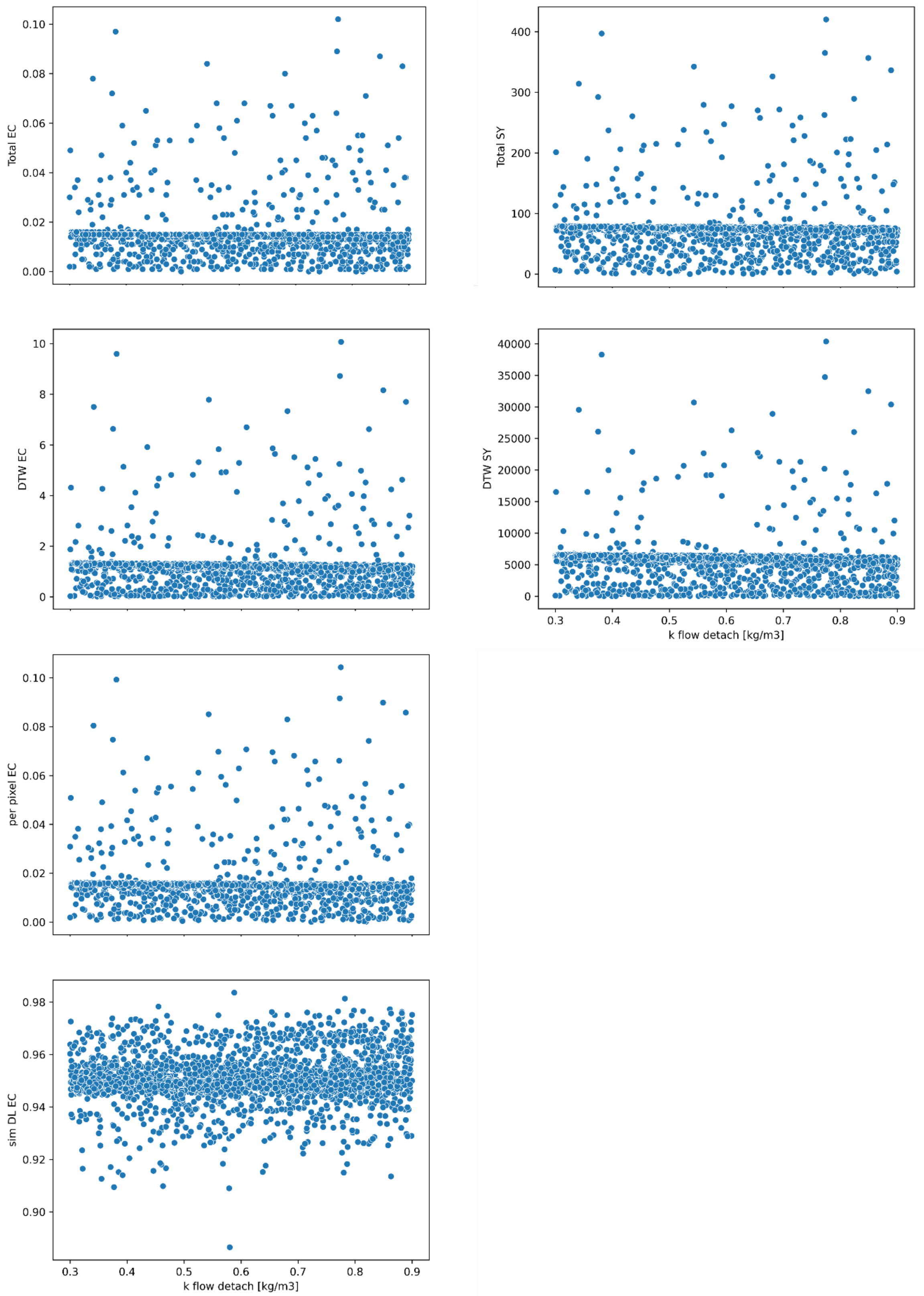

Figure S15: Scatter plot for the laboratory experiment for the model parameter radius of soil shear stress and different metrices of the objective functions.

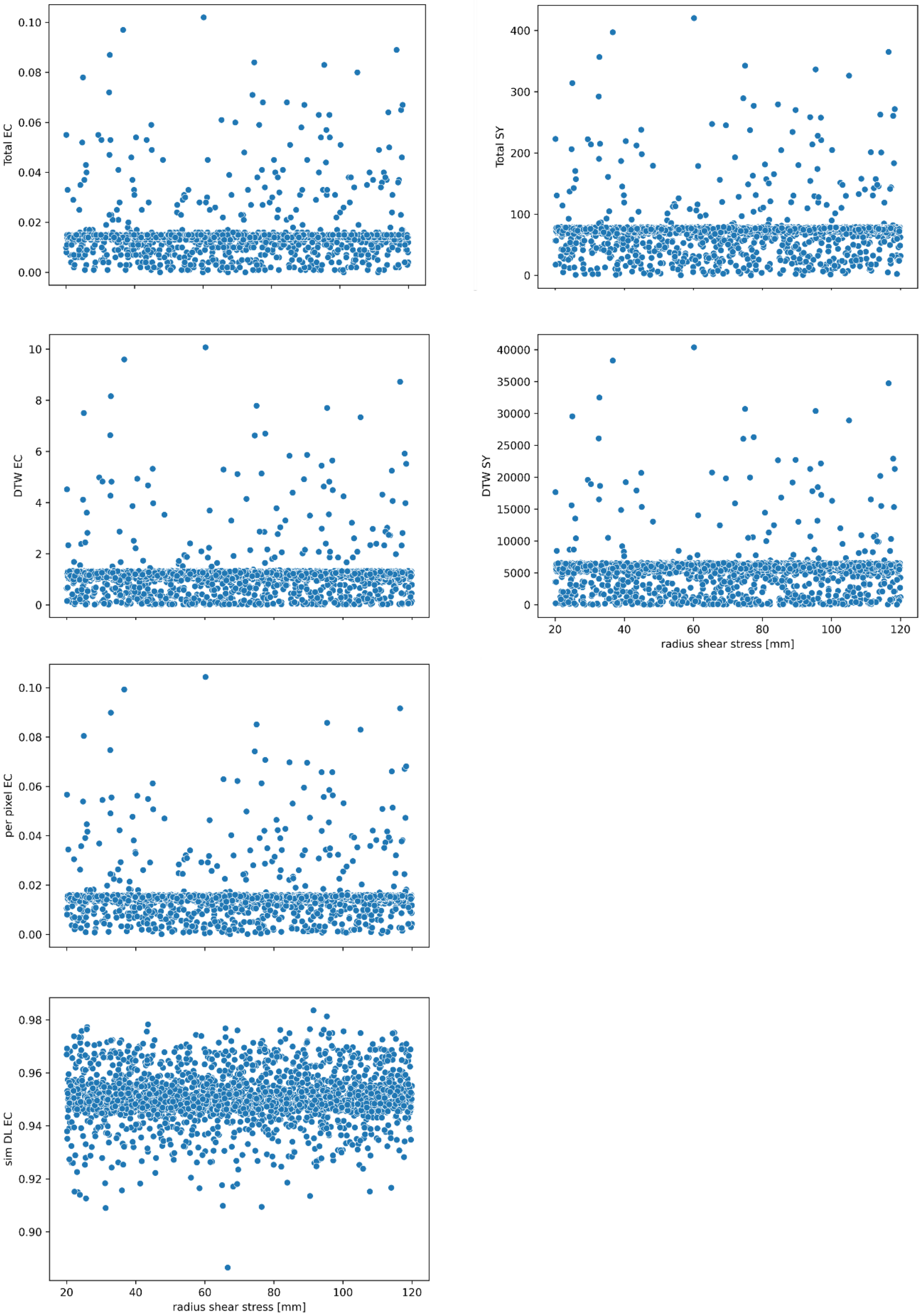

Figure S16: Scatter plot for the laboratory experiment for the model parameter threshold shear stress for slumping and different metrices of the objective functions.

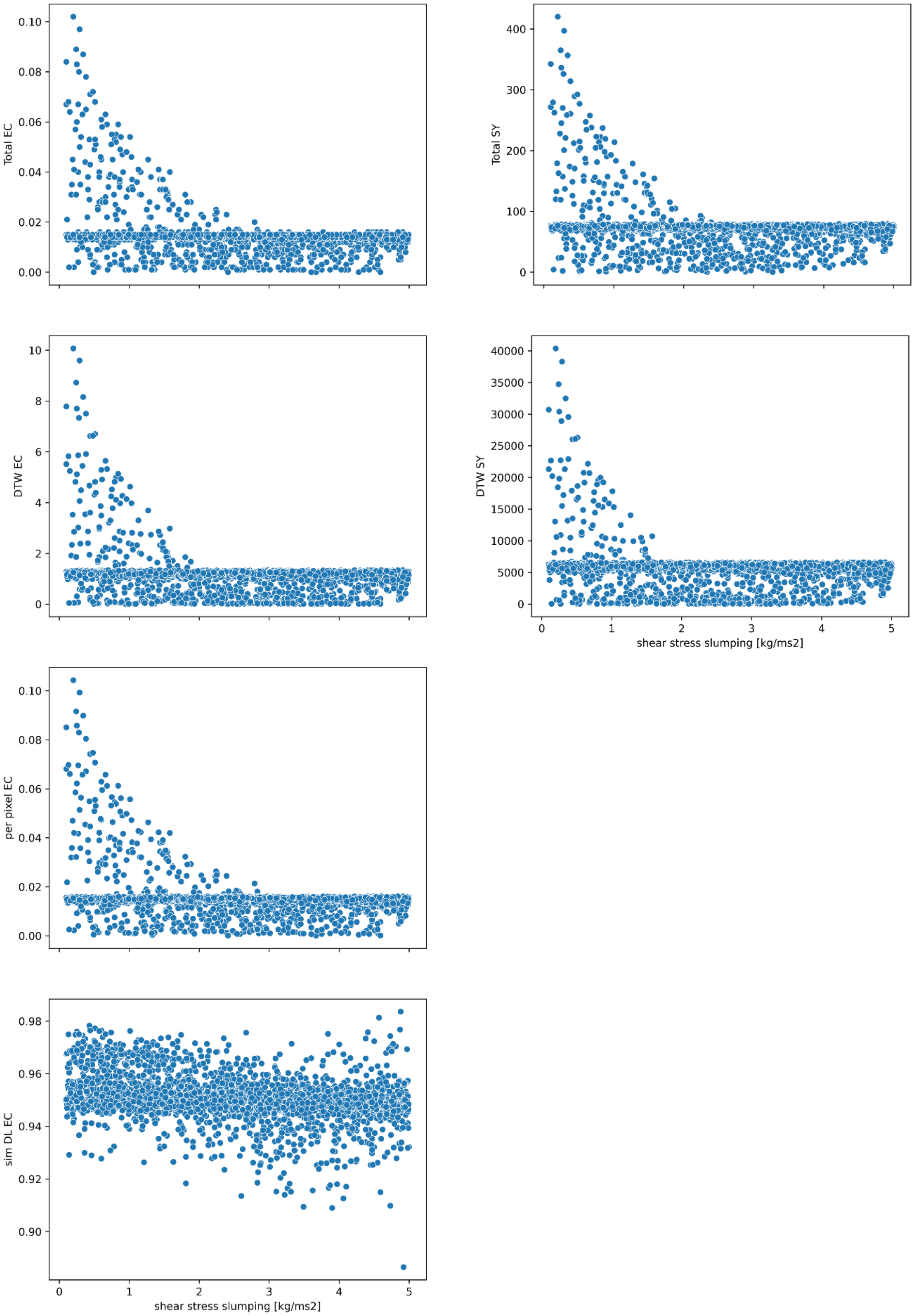

Figure S17: Scatter plot for the laboratory experiment for the model parameter angle of rest for slumped soil and different metrices of the objective functions.